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# The Principle of the Identity of Identity and Not-Identity

József Szigeti

## *The discovery of the new principle: A world-historic act*

In a way that is radically different from the older, precapitalist forms of dialectics, which, as a rule, spoke about the *coincidence of opposites*, just as Nicolaus Cusanus did, or if not in the same words, at least with the same meaning, we find a new formulation of this fundamental dialectical problem in the works and words of Hegel. According to Engels, this new formulation has, as in fact every newly discovered or deeply reformulated philosophical category has, a real world-historic significance as a consequence of the results and proprieties of the unfolding civil society, a fact that would be superfluous to analyze in this context.

The general formula of Hegel is already significantly new. His thesis is not the *coincidence*, but the *unity of opposites*. And the specificity of the Hegelian unity of opposites lies in the fact that the mutually exclusive poles in no way submerge and coincide in an immediate, abstract identity without difference, transcending the manifoldly interpretable unity of empirical facts, as happens in Cue's "*coincidentia oppositorum*"—otherwise very instructively on account of its great philosophical sophistication. Since Hegelian unity sustains components in opposition to each other, in their relative self-independence, the unity becomes *a form of motion*, as an expression and driving force of the eternity of a processing reality—or, to use a more

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adequate time category that conceives eternity not in a timeless manner, but as a subsistence in time as its essence, the general never-ceasing duration—sempiternity, liveliness, motion. Motion is a fundamental category of the whole material world, in contrast to the rigid motionlessness of eternal identity.

Hegelian unity remains in a doubly unambiguous form of motion, even in its manifold subvarieties, in its valid—or even in spite of its invalid—concretizations. (Here we must not forget the silliness and tastelessness of Trendelenburg's and Kierkegaard's widely spread objection that Hegel does not give “examples” in his logico-metaphysical discourses. In fact, Hegel gave examples, sparingly when necessary, even on the highest level. On the other hand, there was never any other philosopher who would have exemplified as he did the validation of his system of categories in the objective world, in his books and lectures on the philosophy of nature, the spirit, history, jurisprudence, and art, not even Aristotle, for the simple reason that there was a lack of a clear demarcation line between philosophy and the mostly non-existent specialized branches of science. He was compelled to treat and sum up much from the latter type of knowledge, which by and for itself is no part of the universal science of philosophy, e.g., most of the topics of the *Historia Animalium*, or the descriptions of the constitution of more than 150 polis, brought together by the members of his academy. They were compelled to assume an obligation in consequence of the undeveloped particular sciences, in order to be able, as “materialistic” empiricists, to realize the analytic-synthetic mental exertion in the interest of a concrete, non-Platonic philosophy.

The self-movement appears in Hegel's theory first as the phenomenon of a self-reproduction *on the same level* as a realization of the lively categorical unity existing not only in our mental work, but outside of it, as the common trait of different phenomena. This kind of self-reproduction may be called “*simple*.” The other is *amplified* self-reproduction, where the amplification is not only a quantitative change—in well-known situations not a positive increase, but a negative one, a decrease, a diminution—but a more fundamental complex of qualitative change to

essential structure. To these viewpoints let us add a third, extremely significant one. Hegel traced a route back from the original *coincident oppositorum*, from this half-empirical, half-transcendent, metaphysical, and in this sense, “mixed” category, to a deeper-lying innermost form, called *Identität der Identität und der Nicht-Identität* [identity of identity and not-identity], playing an important role in the objective and subjective development of humanity, taking the whole conception in the signature of a “concept-realism,” objectifying the subjective spirit almost as much, as subjectifying the objective.

This breakthrough could of necessity only have happened on the philosophical basis of objective idealism and, behind it, in terms of more nearly definable circumstances of the German historical development, determined according to the level of maturity of the given class structure (described in Lukács’s *Young Hegel* and Rozsnyai’s *Revolution and Reconciliation*). And this problem situation was in many respects just as unfavorable as it was favorable, disadvantageous in terms of the fact that it was easy for the antidialectical criticism to burst apart the colorful soap bubbles of dialectical speculation that here and there were lacking a scientifically proven basis, whereas the representatives of antidialecticism, in their earth-bound, myopic empiricism, made the fatal mistake of identifying the never-and-nowhere-seen conceptual dialectical flights of the human mind with a playfully accidental fluttering of the speculative bubbles, advantageous in terms of the given concrete possibility that Hegel could continue his systematic research even in a situation where he had no scientific results to support his dialectical speculations. He was not compelled to break the process of discovery of dialectical connections between categories. He was developing, methodically and systematically, the searchlight to illuminate their intertwined complexity, filling in the gaps with the kind of guesswork, which, in the final analysis, were tentative orientations on reality, although on an insufficiently given, i.e., unfolded and not sufficiently clarified, reality.

To create this new theoretical situation, Hegel had not only to take into account the earlier appropriated forms of dialectics, but

also the serious problems of the fundamental laws and principles of formal logic. According to the predominant conception of his time, they were the fundamental constituent categories of the whole field of formal logic, i.e., of concept, judgment, and inference, and not without competence and power. Today, if this problem is viewed from the standpoint of mathematical logic, which rules almost everywhere even as the organon of philosophical cognizance, all that seems to be completely out of date. In this case, however, precisely the relatively undeveloped, elementary character of the problem seems fruitful. This happens also, more than once, in other fields of scientific research. The uncomplicated character of the problem makes it easier to comprehend, just as in the same manner the successful investigation of the simple, general process of combustion made it possible for Lavoisier to demolish the hitherto ruling paradigm of the phlogiston conception—leaping over the varieties of forms of thermal motion, the analysis of which had been made possible to a certain degree by his oxidation theory.

***Dialectical, formal, and formalized logic. Mathematical logic as a partial field of abstract algebra***

The theoretical springboard of the previous question is the fact that different things are not subjected to discrimination; the very different spheres of formal logic and of formalized formal logic are treated as if they were the same. And this is by no means true. The mistake conceals the historical fact of scientific discovery mentioned earlier, notwithstanding the fact that formalization examines and treats not formally conceived logical structures, but the more abstract, the last remnants of intensionality extirpated, structural connections and complexities, unimaginable to formal logic, having essentially the signature of modern abstract structural algebra. Its primordial theoretical ground (as a basic ontological cause and fundamental logical presupposition)—in spite of every rectification and ramification, e.g., many-valued logic, or modal logic set as an aim of algebraic formalization—remains Boole's binary, two-valued algebra. As its consequence, algebraic logic does not so much make a logic

from algebra as an algebra from logic. And this is more and more obvious today, when that type of criticism of philosophical dialectics and dialectical philosophy that axiomatically presupposed as its legal ground and mainstay the mistaken identification of philosophical methodology with mathematical logic, belonging in the domain of abstract algebra and not to philosophy, becomes unacceptable. The false premise about mathematical logic as the true (scientific) organ of philosophy has been critically discussed many times and with the use of ingenious arguments. Even in that citadel of the interesting and the creative, the Massachusetts Institute of Technology, which has produced such useful constructions, this viewpoint emerges most plastically perhaps in a study by one of its members, the mathematician Gian-Carlo Rota, "Mathematics and Philosophy: The Story of a Misunderstanding" (1990).

In spite of that, the general situation is still characterized by the fact that the *farical* thesis about the perfect irrelevance of fundamental formal-logical laws concerning the logistic universe of discourse, can still be propounded with a certain scientific cogency, because a misleading view still prevails that the *principium identitatis* or the *principium contradictionis* (this may be much more aptly termed, using the French tradition once known for its precision of diction, the *principe de la non-contradiction*) are *pur et simple* derivable from the *calculi*, especially from the conjunction.

A specialist must be very unobservant, and betray a sorry incapacity for conceptual mediation, if he or she does not take into account the fact that any form of the calculus from its very first appearance already presupposes both principles. They immediately presuppose the exclusion of contradiction and at the same time the *internal* operation of negation in the conjunction, by means of which they try to prove them. As a matter of fact, the "and" of the conjunction (its symbol is " $\wedge$ ") is true if and only if both of its conjugates ( $p$  and  $q$ , called variables) are true. But this is by definition (i.e., a consequence of their undeclared presupposition) impossible in the case of  $p$  and not- $p$  (or taking  $p$  with the negation sign " $\neg$ " in the formula " $p \wedge \neg p$ "), because at



the very beginning, at the demarcation of the conjunction, they have taken as a starting point for an absolute untranscendable truth the principle of *abstract identity* and its quasi-correlate, the *exclusion of contradiction*.  $A$  is always equal to  $A$ ;  $A = A$  and can never be equal to its own negation  $\text{not-}A$ ,  $A \neq \neg A$ . These are eternal relations.

From the dialectical point of view, the *autodynamism of negation* would pass inevitably to its accomplished form, the “*negation of negation*,” as we shall prove by the necessity of reflexivity. This is expounded in its insurmountable theoretical sharpness and generality in Hegel’s principle of identity of identity and not-identity. The formalists bring the movement of negation to a deadlock by means of the absurd contradiction—seemingly absolutely unsolvable—interpreting every kind of contradiction as absurd. They get some profit from their preference. Their insufficiencies, problems, and self-contradictions emerge when they make absolute the limited validity of their narrow principle, and this small-minded, illiberal notion becomes the measure of everything, in the case of beings, whether or not they truly exist; in the case of concepts, whether or not they are true; in the case of dialectics, whether or not they are scientific.

This “ingenious” proof using conjunction as a whetstone falls into the logical trap of the vicious circle, presupposing the theses that are to be proved. The spontaneously accepted *a priori* formal-logical principles determine that in every permissible value-distribution they must inevitably be false. Applied like this, however, this is no proof at all, but simply a symbolic display of that which they mentally presupposed. It is shown in tabulated form by the so-called *truth matrix*, the rectangular array of elements in rows and columns, treated as a whole. Representing a truth matrix, I write the truth or falseness of the calculus constant describing its characteristic value-distribution [*Wertverlauf*] or configuration with the upper-case symbol “ $T$ ” or “ $F$ ” as opposed to the value-distribution of the variables written with lower case “ $p$ ”(rue) or “ $f$ ”(alse). I give first the conjunction of  $p$  and  $q$  and then that of  $p$  and  $\text{not-}p$ , and twice for the

sake of symmetry with the previous, and immediately in the value of negation (which we will do in other connections too), taking the value of  $p$  as  $f$  in the repetitive part of the matrix:

$p \wedge q$	$p \wedge \neg p$
$t T t$	$t F f$
$t F f$	$t F f$
$f F t$	$f F t$
$f F f$	$f F t$

Our matrix clearly shows that the value-configuration of  $p$  and not- $p$  is *tautologically* always  $F$ . And it cannot be anything else if the conjunction is an operation based on the principle of abstract identity remaining its absolute measure. The negation is perfectly excluded from its internal structure even as a potential agency. It is brought in only from without, externally, as a quasi-correlative factor, to play its nonnegligible role. Hand in hand with this goes the devaluation of contradiction, leading to a logical deadlock where no progress can be made. As a consequence of this rigid unconditionality, their spokespersons find themselves unable to conceive a true idea about the real difference and contradiction between formal-logical and dialectical contradictions. Although the former can be in no other way understood except by dint of the latter, just as formal logic in general is a borderline, externalized, marginal form of the dialectics of inference, assertion, and concept, as I outlined in part 4, chapters 1 and 2, of my *Revolution of Scientific Thinking* (1984),<sup>1</sup> and which I will go on to prove at the appropriate juncture at the key places in this essay.

Though abstract identity, as a measure presupposed by conjunction, excludes from itself the internal character of contradiction, conjunction, as a specifically mathematical-logical, i.e., abstract algebraic configuration excluding contradiction, does not exclude negation in the same rigid way.

In a certain way, both in terms of the true and false values and also in their relations to each other, negation even comes into a more intimate connection (to use the expressive word of Quine) with it than does formal logic, which is interested only in

the correctness of thinking, not in its truth, to such a degree that without negation neither the conjunction nor any other of the sentential calculi could have been defined. In this respect, without mathematical formalization, mathematical logic implies, much more than the formal logic, the decisive, liquefying, and forward-driving categories of dialectics. The problem of determining from where do we get the negation, however, remains.

Our knowledge of the answer to this comes from dialectics and by no means simply from the everyday language, as has often been asserted. But when we speak about this more intimate connection of mathematical logic with negation and dialectics, then we must be well aware of the characteristic property that as a rule this connection does not always and necessarily appear in an immediate form of its dialectical being by and for itself.

This theoretical sphere of thinking, in the final analysis, is the sphere of modern abstract or structural algebra. I use this latter term in the sense of its more clarified meaning that began with the epoch-making work of E. Steinitz, according to which it now means the diapason from the number field [*Zahlkörper*] to any field [*Körper*] taken at our discretion. This sphere is so much intertwined with negation that the principle of double negation—Sheffer's stroke, "neither . . . nor"—is the only calculus by means of which we may express all other calculi, at least if we dismiss Quine's opinion about the functional equivalence of joint denial and alternative denial with regard to the equal reducibility of every other calculus to either of these. In this respect, our viewpoint is that we must unconditionally give *priority* to joint denial, the first and unique place, historically as well as systematically and logically, rather than to alternative denial, for the reason that its symbol contains quantitatively smaller and qualitatively stronger mathematical ideas behind its symbolic character mask than the other—which thesis must still be proved in the theoretical expositions that are to follow. It cannot be a fortuitous fact that Peirce, anticipating so much that would be significant from later development, discovered joint denial as early as 1880, whereas alternative denial was demonstrably discovered only in 1902. It is true that Sheffer discovered both in the same year (1913) without knowing anything about Peirce's

findings. This took place, however, on a much more developed level of formalization.

***The comparative analysis of the autodynamism of dialectical negation and joint denial***

The negation of joint denial, which I call in its constituent elementary form twofold negation or *double negation* (for reason of its affinity to Marx's term *Doppeltsetzen*), where I do not make the fatal mistake that they have the same conceptual content, and still less an identity with "negation of negation." The variables are simply in no way connected to each other, neither in this elementary form, nor in the extended manifold form, by means of the conjunction's constant connective "and." There is no inevitable "and" between the two or more negations, just as little as there is no necessity to use *and* in the linguistic formulation of the "identity of identity *and* not-identity." The ideational content may be adequately or still better expressed using other linguistic formulations (e.g., "identity of identity with not-identity," where we take for granted that there are profound differences between the words of conjunction, and they cannot be reduced in such a context to a single "and"). Ample proof of this follows.

In Hegel's fundamental formula, the second member of the given relation, not-identity, owes its *prima facie* existence to the positive starting point of the identity that will be negated. It is not one of the related poles of a dualistic relation, as in the relational logic "here and yonder." The two things are not even correlative; their relation is not a co-relation like the mutually co-implicating notions of "here and there" or of parent and child, because the starting identity, marked off from anything else, takes upon itself the moment of negativity, but only implicitly wrapped up in the deceptive appearance of absolute positivity. And this deceptive appearance has such an amazingly tenacious hold on the history of thought that the pseudoabsoluteness of positivity adheres to it, even after Spinoza's penetrating and profound analysis of the dialectical principle *determinatio est negatio* (1895, 361).<sup>2</sup> (Spinoza's letter was originally without *omnis*, which even Hegel mistakenly included with it, although

there is a tremendous difference between the two formulae.) In this way the implicit negation does not become an explicitly asserted negation until some time after its discovery. On the other hand, the identity is asserted from the very beginning as absolute positivity, without any implication. There is nothing before it. Or, if you prefer, nothing is before it, except in a disguised form, as something that Fielding already knew and developed in his witty and jokingly sociocritical *Essay on Nothing* (in his *Miscelleniae*). And the disguised form, the nothing-that-appears-to-be-something is none other than the internal, not external, implicit negation constituting and delimiting the thing itself. It must develop explicit negation to realize concretely this twofold unity, or to use Brouwer's term *Zweieinigheit* (1975b)<sup>3</sup>—or the Russian term *dvoiyedinstvo*, which was very frequently used in Soviet times. As negation becomes explicit, it must, in its self-explication, accomplish all its movements fully, in order to reach a truly concrete identity.

What does this “accomplish all its movements fully” mean? It means a great deal and is highly significant. If negation negates everything except itself, its power and effect remain restricted and relative, in a one-sided external direction. It loses—as Hegel put it—all its “dreadful power,” being outside of itself. In order for it to become absolute, its power must be internal, it must negate everything, including in this everything itself, that is, the negation. The negation intending itself is reflexive activity, or rather movement. It is a bending back on itself, to the negated self, to not-identity. The negation accomplished in such a way cannot be anything else except the *negation of negation*. With the negation of negation, with this accomplished development, we attain first the intellectual position being able adequately to conceptualize the relative component of the negation's absolute nature, the conversion into a *synthetic identity*. This synthetic identity, when expressed in German (or English), comes first: *Identität der Identität un der Nicht-Identität*. In Hungarian, it comes third and last: azonosság és nem-azonosság azonossága (identity and not-identity's identity). In these expressions the German and English are quasi-analytic, i.e., unfolding something

already from the beginning as synthetic unity, whereas the Hungarian is quasi-synthetic, summing up in the final identity the previous analytic moments. Both are, however, only *quasi* one or the other, because the ideas contained in them are just the same. In them, such a synthetic identity plays its part on a higher level, just linguistically in different places, in which the negation emerges to the light as the driving force of the closing stage of the process of development, conserving the substantial results of the stage of their differences.

Another possibility exists, however, seemingly completely different from what has been indicated. This should be called *regressive metamorphosis*, the turning back after, or more accurately *in*, the second negation from the starting point. A great number of logicians—from the brilliant DeMorgan and Boole to the logical empiricist Karl Popper—interpreted this as a simple falling back to the very beginning, without any result. In this, however, they made at least a twofold mistake. First, they abstractly identified this form with a mathematical proof, *reductio ad absurdum*, which is in fact more deeply and widely embedded in the dialectical process of regressive metamorphosis. And even in such a case, it is correct—but not always correct—as the reduction process is a mediation. After negating the starting thesis *A*, they begin to give proof of  $\neg A$ , coming in this way to an absurd contradiction, absurd because it is unsolvable, i.e., it does not grow into a higher movement. The result negated (second negation) compels us to return to the starting thesis *A*, which is by no means the very same as it was at the beginning; it is a much stronger *A*. This proves the correctness of the original thesis. But not always; only in a restricted sphere. Brouwer's (partial) elimination of the principal of the excluded middle or third term was grounded on the fact that as a rule there is frequently a mediating median [*medius medians*] between the two poles, which alone clearly demonstrates the more complex character of this problem sphere. In this context, what must be sufficient for us—just to put the emphasis on its original ontological character—is the important remark that regressive metamorphosis usually appears as a crucial phenomenon if the

self-reproduction on a single level begins to stagnate, and the adequate reflexivity does not unfold itself in time as a driving force—not to speak of the case when it is entirely inadequate, as in the catastrophe of “existing socialism.” In light of this short statement, we may perhaps see and feel the whole significance and even the diapason of the problematic of regressive metamorphosis.

The double negation of joint denial cannot not be regarded as negation of negation because it lacks self-reflexivity. It denies the truth or falsehood of a concrete thing that is not itself. The two negations are loosely, extraneously connected in the necessity of rejection of some factual situations, or, to use the decorative name, their “truth-value.” They do not lead us to a positive truth as the synthetic identity of the negation of negation, or, among others, to such forms of judgement and inference, e.g., *remotive judgment*, which are the further consequences, positive and categorical conclusions of the concrete negations. (To illustrate: The sum of the angles of a triangle are neither smaller, nor equal, nor larger than the sum of two right angles. The sum of their angles is a function of the prevailing spatial relations, of the Bolyai-Lobachevskian, Euclidean, Riemannian, i.e., of the hyperbolic, parabolic, elliptic spaces.) The reason for this situation is that in the dialectical forms of ideas we move of necessity to the concepts of a concrete whole; the negation of the specific articulations of this whole will therefore take us up to that height of the universal that essentially contains in itself the capability of adequate concretization (in our example, functions of spatial relations, instead of space in the abstract), from which we may descend to the specificities of this relationship. The identity of the categorical judgement in this way cannot and must not be abstract, but concrete. Add to this that the descent from the universal to the specific does not necessarily mean a step from genera to species, as it does quite consistently in Aristotle’s closed worldview.

The negation of joint denial becomes true if both variables are false, the negation of falsehood being truth. The variables of the calculus are semantically either loosely connected or not

connected at all. If joint denial is taken in this semantically indifferent form not involving “and,” it must have the systematic priority concerning *alternative denial*, which is often taken as its functional equivalent. The functional equivalency means that only these two denials are the appropriate means to substitute all the other connectives of the sentential calculi. But this is, I am afraid, a rather superficial view. I do not believe that there is any such functional equivalency between the two types of denials. In the second type, alternative denial, the alternation is absolutely necessary not only in name but also in its deeper structure, whereas we do not need to suppose an “and” either in the structure or in the name of double denial. And this means that the first is a more complex, compounded form compared to the relative unity of the second. If we want to get a *primordial* connective that will reduce all the others to this, then it is perfectly sufficient to have only one. Two would be too many; one of them would be superfluous and could not be *primordial*.

If we take a closer look at the whole problem, we will perhaps come upon an important as-yet-unilluminated trait. The usual symbol of double denial is an arrow pointing downward, “ $\downarrow$ ”; that of the alternative, a separating straight section, “ $|$ ”

Putting their matrices next to each other, we get a graphically descriptive picture of them by setting their (truth) value under the variables and connectives:

$p \downarrow q$	$p   q$
$t F t$	$t F t$
$t F f$	$t T f$
$f F t$	$f T t$
$f T f$	$f T f$

In the double denial, *F*(alse) is dominant and *T*(ruth) is recessive; in the alternative the reverse is the case. The loose connection of *neither . . . nor*, or in a more complete form, *neither this . . . nor that*, or still more complete, *neither p is not t(rue) . . . nor q is not t(rue)*, may be expressed by a simple reduplicating, *double-setting* (Marx’s *Doppeltsetzen*) of *not*, putting it side by side: *not this, not that, not p, not q*, or rather: *p is*



*not t, q is not t.* Or in the case of the truth of this denial, *p is not f, q is not f.* In this simple form, even the possibility of a substitution of “and” is excluded. This brings us very near the supposition that we shall consider in another context, that the real primordial form, the root-ground *primitive*, must be the negativity, *not* in a *restricted autodynamism*. Otherwise the double negation is nothing other than “not *p* and not *q*,” ( $\neg p \wedge \neg q$ ). But in this formula we now have two logical constants ( $\neg$  and  $\wedge$ ). In this way it cannot be the primordial primitive for which we are searching.

On the other hand, alternative denial, which many specialists take as the functional equivalent of the previous connective—even the outstanding W. van O. Quine—shows *prima facie* its superfluous nature in at least two (but in a later phase of inquiry we shall show three) mathematical ideas of connectives, integrated not only in its designation, but even in the case of two, quite extraneously, in the constant self. Since the logical constant expresses that *either* this is *not* true *or* that is *not* true, *or either* this is *not* true *or* that is *not* false—an admission that both may be false—therefore negated, this must be true as well. Taking all these together, we may define the sentential calculus of alternative denial: it must be true if *at least* one of its variables is false. In this case the negation of this false variable is *principium sufficiens*.

***The synthesis of heterogeneous components  
in the formation of mathematical concepts***

In the formation of mathematical concepts, this kind of synthesis is quite frequent. It well-suited to the definition given earlier, with *at least* included in its structure. Such is, e.g., the triangle inequality, taken in its geometric form, the thesis that the sum of the two sides of a triangle is at least equal to the third,  $a + b \geq c$ . This *at least* incorporates in itself two radically different instances, being heterogeneous. The first,  $a + b > c$ , is absolutely in order. But the second,  $a + b = c$ , is not only not in order, but directly diabolical! A triangle with a third side

measuring exactly the sum of the two others does not and cannot exist in visual Euclidean space. Such a triangle would have to be a straight-line section, which seems perfectly absurd.

But nothing is purely visual, or bound merely to our senses in the world of thought, not even in sense of undefined intellectual vision. In fact, the way we think today has left the starting point of sensuality and visuality far behind. Kant's subtle idea maintains its validity only concerning the elementary foundations of our thinking: "Thoughts without contents are empty, visual perceptions [*Anschauungen*]<sup>4</sup> without concepts are blind; we must therefore sensualize our concepts (i.e., add to our vision the sensory object) as well as make our visual perceptions rational (i.e., subsume them under concepts)" (Kant 1877, 126).<sup>5</sup> How can we follow Kant's advice in the case of "spaces" having more than three dimensions, whether "Euclidean" or not, none of which are visual?

Conceptually, we can easily solve the (geometric) difficulties, posed by the triangle inequality, where we also take into account the sensory-visual element. It is not necessary to look to other related connections for help, claiming, as some leading mathematicians do, that if our supposition were not this "*at least*," then we would be in serious trouble. This is, however, not a solution to the problem; it is only a way of putting it off indefinitely, in the ultimate analysis, an escapist, positivist attitude, distancing itself from philosophical and dialectical problems in the spirit of an unconditional mathematical autonomy, proclaimed, not without reasons, by Hilbert and followed in this point by Brouwer, with more self-contradictions. The triangle with its three closed sides falling into a single straight line is the configuration of a triangle either just beginning to come into existence or in the last stage of degeneration, in the final result no more than the two-fold unity of two straight lines lying within each other, fixed on one of their ends, opening on the other to create a third side, or to annihilate it. By implicitly mentioning temporality as a process, we reproduce reality, let us say, in the process of drawing a triangle with a crayon. But this temporality may be regarded

metaphorically too, if we restrict ourselves to a genetic definition of the triangle, where all the dialectical movements seem to be a spaceless and timeless connection of pure concepts.

Let me include here a remark about the complement of the analyzed form. If there is such a synthetical connection between greater and equal, such as *at least*, then there must be a similar connection between less and equal expressed by *at most*. There are a great many varieties of such syntheses. In projective geometry, the point is used in cardinal connections as the *point of intersection* of a set of straight lines. What is more, the “*ideal point*” (to characterize the problem in an elementary way) is the fundamental constituent category of the total sphere, enabling us to extend well-known basic operations. It enables us instead to declare that straight lines either intersect each other or they are parallel, a new synthesis being created by means of *at most*. The new assertion must be that straight lines always intersect each other, *at most*—being parallels—at an infinitely distant mutual ideal point. Geometers are not disturbed—and they must not be disturbed—that geometric theory is built up in a unified way on the basis of the set-theory of points, and by their new method, by definition of the points as intersections; the point as the ground category becomes a consequence, a derivation of other relations. A deeper-going, more extensive analysis could have shown that the ultimate fundamental relation remains the point.

It is interesting to see that historically those mathematicians who have studied a dialectical philosophy knew much about the fundamental significance of the point, as did the mathematician Cavalieri, well-informed in Campanella’s dialectical and communistic philosophy (communistic in the premodern sense, which takes as its starting point not production, but what is akin to distribution). Cavalieri was one of the pioneers of higher analysis with the conceptual apparatus of *indivisibilia* (=ατομος). Thomas Aquinas, on the other hand, tried to apply this (originally Aristotelian–neoplatonic) concept to “incorporeal substances.” “Indivisible is what is out of the whole genus of continuity, and in this way out of genus of incorporeal substances as God, angels, souls are denoted indivisibles” (*Sum.*

*Theol.*, vol. 1, part 1, question 8). Cavalieri applied this to the continua of geometric figures, neglected by Aquinas, “creating one of the most influential books of the early modern period, *Geometria indivisibilibus continuorum*, 1635” (Boyer 1968, 361). For him, as for his philosopher, Campanella, the point represented a contradictory unity, a spatial extension without extension, a *null-dimensional figure*. (The expression in italics conceals a contradiction, because dimension = measurable extension of any kind, and how can zero be measured? Any bumbling attempts by those who use the expression without the dialectical concept behind it are useless; they would not have the slightest idea what depths are under their feet when they tread on the words of this term.) The line—a straight line—represents the movement out of itself of the self-negating point, arriving at its own accomplishment in constituting the new figure, a one-dimensional space. It cannot be a curved line, only a straight one, because curvature already presupposes something else, namely, a two-dimensional space, a plane. The plane comes into existence by self-negation of the line, as a true, concrete continuity of the points. By concreteness of continuity is meant that its oppositional moment, the discontinuity, is contained in it in potential form. The plane must now step out of itself, bringing into existence a three-dimensional continuum, a real space. When in modern geometry the three-dimensional continuum steps out of itself, the  $n$ -dimensional ( $n = 4, 5, 6, \dots$ ) space continuum is the stepping out of itself of the  $n-1$  dimensional. The “spaces” above three dimensions are just as much abstractions of real space as those below, with the difference that the former eliminates the sensory nature of a real space much more radically, without losing its applicability to other sensory phenomena.

Three-dimensional space, by the productivity of a not-quite-clearly conceptualized negation of negation, is here reduced to a set of zero-dimensional (“mathematical”) points, with a regressive metamorphosis stepping backward, like a crab, eliminating the higher dimensions one by one until there is nothing left to eliminate. In opposition to this regressive deconstruction, the other necessary way of looking at the question of progressive

reconstruction has its basis not in abstracting (*abstrahere* = to take away, peel off) negation, but in concretizing (*concrecere* = to grow together, develop into a unity) negation, ceasing and preserving (*tollere et conservare*) something on a higher level. The destruction and reconstruction, the analysis and synthesis, the downward and upward, was in the earlier-mentioned case of Cavalieri's theoretical exposition of space dimensions a way of thinking, a theoretical thought. But is the dialectical course of our thinking in this case adopted to the movement of reality, or does it differ from it completely? It seems to me extremely probable that it cannot be adopted to the world—the whole world—despite the almost unanimous assertions of the proponents of the big bang theory—though it would be easy to identify an absolute beginning of the world with such a “creative” act. It could even be seen as a triumphal victory of dialectical thinking. I am not, of course, afraid of a materialist dialectic. but we would at once be entering a maze of unlovable absurd contradictions, marching triumphantly from Erewhon through Erewhon to Erewhon, to use the word of our modern Samuel Butler. A smooth and simple way of thinking is not necessarily a guide to truth. I think it more probable that the question of deconstruction and reconstruction is more intimately connected with the degeneration and genesis of the partial systems of the universe, even when these processes continue in the real space-time continuum.

Both analyzed connections—degeneration and genesis, as well deconstruction and reconstruction—are used in the specialized branches of sciences, usually together, not always with the same adequacy. This was the situation with the two primordial disjunctions. According to Quine, alternative denial has the same effective power, and is just as good a way of reducing the other sentential calculi, as double or joint denial. We have seen, however, the “or,” although not explicitly realized as a logical constant in the formula, is unquestionably an integral part of alternative denial, whereas “and” is not an integral part of double denial, at most a very distant and negligible reflex of “also” and “as well.” The “not  $p \vee$  not  $q$ ,” in its new formalization

$$p \mid q \equiv \neg p \vee \neg q$$

is a more complete and therefore more adequate expression than double denial would be in a conjunction that denied both variables ( $\neg p \wedge \neg q$ ). From a formal point of view, the fault with ( $\neg p \vee \neg q$ ) is that it has more connectives than are necessary for a primordial.

***Symbol and conceptual content. Their relation to functional and substantial rationality***

Have we already exhausted the complete ideational content of our symbol “|”? Do we already know what conceptual elements are synthesized in it? Hardly. I believe that further inquiry is necessary. Taking the relevant fact that there are two kinds of disjunction, the exclusive and the inclusive, we can take a step forward. A new name, *contra valence*, has recently been given to the exclusive to differentiate it terminologically from the inclusive. The term *alternative* is strictly related to this terminological range of problems. What is the meaning of the term *alternative* in alternative denial? In earlier centuries, by far, the most frequent meaning of *alternative* was a choice, a decision between two or more mutually excluding possibilities. Truth does not always exclude false, nor false always exclude truth, in terms of formal logic. True alternative must be, with the power of formal logic, a choice, a decision between false and true, or in the reverted position of the antecedent and consequent, between true and false. The ground of alternative is exclusive denial, whereas the term *alternative denial* uses it in the opposite sense. This terminological hurly-burly necessarily intrudes into the background ideas, too. Putting exclusive disjunction (or if you prefer, *contra valence*) into matrix form, with its value-distribution symbolized by the upward-pointing arrow (“ $\uparrow$ ”), we get the following picture:

$$\begin{array}{ccc}
 p & \uparrow & q \\
 t & F & t \\
 t & T & f \\
 f & T & t \\
 f & F & f
 \end{array}$$

This  $F-T-T-F$  value-configuration is totally different from that of the inadequately termed alternative denial,  $F-T-T-T$ . The latter has the same value-configuration as the *inclusive* (*vel* in Latin) *disjunction, set with negated variables*. Inclusive disjunction without the previous restriction is the generally accepted form of disjunction. The new name for exclusive disjunction, *contravalency*, may be the sign for this situation. It would be advisable—as a way of putting an end to the terminological hurly-burly—to rename alternative denial *disjunctive denial*, as one of the modes of clarifying our conceptual thinking by making adjustment to the symbols and their meaning, which would also provide more self-confidence and assurance of the—always only relative—freedom from contaminations of our theoretical operation.

Leibniz already knew about mathematical symbols (where the same symbol may be expressed by different signs as the history of number-writing with different ciphers shows); they are abbreviated representations of highly complicated conceptual connections, relieving our minds of the burden of having to go through the same labyrinth of thoughts again and again. And Whitehead, speaking about mathematical symbols, says much the same:

In mathematics, granted that we are giving any serious attention to mathematical ideas, the symbolism is invariably an immense simplification. It is not only of practical use, but of great interest, for it represents an analysis of the ideas of the subject and an almost pictorial representation of their relations to each other. (1948, 40)

The enormous benefit reveals its more or less inevitable unfavorable feature when the technically self-assurant, or at least appropriate, operations with symbols have no really clarified conceptions on some critical points, on some parts of the essential components. Outstanding mathematicians, famous didacticians of their discipline, for example George Sheffer, whose *Lehrbuch der höheren Mathematik* was repeatedly published in amplified and modernized editions throughout the

twentieth century, made more than one mention of his experiences with university examinations and colloquia, in which technically well-equipped students were unable to give a clear theoretical account of the fundamental principles of higher analysis. The mechanical operational activity seemed to make knowledge of the deeper-lying conceptual movements redundant.

To a certain extent we may understand Whitehead's point of view, however, when, after the quotation just cited, he continues with the following idea:

It is a profoundly erroneous truism, repeated by all copy books and by eminent people when they are making speeches, that we should cultivate the habit of thinking of what we are doing. The precise opposite is the case. Civilization advances by extending the number of important operations which we can perform without thinking about them. Operations of thought are like cavalry charges in a battle—they are strictly limited in number, they require fresh horses, and must only be made at decisive moments. (61)

But—to concretize Whitehead's metaphor a little bit further—the larger and more valuable part of all that which today is an everyday mechanical element of tactics was at one time the result of heroic Balaclava charges. The trouble does not begin when we fail to map out step by step every intellectual presupposition of our operational activity with symbols that would lead to any kind of quick and effective action. The trouble begins when we have no idea about them, just when the theoretical presuppositions would have been indispensable. The technically well-equipped students do not know it since they never understood them. Not only can they not put this set of ideas into their memory, they are also unable with their mechanical thinking to discover it at all. They never acquired the kind of thinking that distinguishes a superficial language game from substantial material knowledge, absolutely necessary to carry out the kind of formal thinking that is rich and elastic enough to grasp everything that can be grasped. The proselytes of a hard and rigorous division and



opposition of formal and material culture, in their pedagogical options, make instruction unproductive, being unable to recognize that the alienated forms of formal and material culture are extraneous states, because neither this nor that phenomenon is what it should be. The form is not an authentic form, as they imagine it is, because authentically it must be a concrete form with content. The material side is not what they believe, i.e., a content, because the authentic content is a material substrate already formed concretely into a substance.

It would be really dreadful if everybody had to know, let us say, all the electromechanical devices necessary to get to a higher floor, instead of simply knowing how to press the proper button. But it would be even worse if an electro-mechanical specialist's knowledge extended only to the pressing of buttons without any knowledge of the scientific presuppositions that make the entire device, the elevator, a new and quicker way to ascend. This is the reason why philosophers and sociologists—to follow in the footsteps of Max Weber—very properly differentiate between “functional rationality” (pressing buttons) and “substantial rationality” (knowledge, application, and development of operational principles). All this seems relatively to be a truism. But the labyrinths and pitfalls of modern scientific and technical progress keep on creating again and again situations where things that are coordinated will be with the outmost rigidity *mechanically* disjoined and things that have nothing to do with each other will be identified in a mystical *organic* way. This kind of mechanical disjunction and mystical organic conjunction makes a critical demystification a matter of intellectual integrity and responsibility. The real state of things must be demonstrated. In the case of our problems, the real relation between symbols and their conceptual meaning, their connection in the relative totality to which they belong, must be shown.

***The inclusive disjunction “or” as synthesis of the dialectical poles of exclusive “or” and conjunctive “and”***

Alternative disjunction, or contravalency, differs from the inclusive disjunction; it does not give any possibility of choice or

decision between the same truth-values, between  $t$  and  $t$  or  $f$  and  $f$ , because they are identical. In inclusive disjunction, on the other hand, there is the possibility of a choice in the identity. This must be a “mixed” form built up from the alternative and its dialectical opposition, the conjunction. Inclusive disjunction holds true if this or that state of affairs [*Sachverhalten*], or sentences “ $p$  or  $q$ ” are true, but both may be true as well. The similar usage of standard, everyday language, which logicians, so to say, codify, usually serves as proof. They are seldom interested in the serious question of what movements and categories are hidden behind the easy flow of the everyday language, being too much accustomed to the false—at least partially—fictive identification of logical and grammatical syntax. But the contradictions arising as a result of this ambiguous position compel them on the one hand to turn from the “natural”—in reality sociohistorical—languages to the artificial construction of an ideal language, while on the other hand the mechanical rigidity and gaps of this construction often send them back to the uniquely fine solutions of the “natural languages,” which are, from the logical point of view, often relevant. We must understand the close connection of “or” and “and” as an interaction and spontaneously dialectical unification of the real poles of our thinking that, because of the spontaneity in usage, regards it on the one hand as a linguistic factum, while on the other hand some logicians try to impose barbarically—to use Quine’s disapproving word—the clumsy expression “and/or” on the living language, both spoken and written, being dissatisfied with the context-conditioned general acceptance of the expression in the immediacy of practical language.

If contravalency and conjunction are really united in the inclusive disjunction, then alternative denial is of necessity a synthesis of not only of two, but three, connectives.

It is easy to visualize this by giving a “proof,” but we must present the formalization in a somewhat unusual way. I shall put down all three as follows. On the left is the matrix of the alternative, on the right the conjunction. Between the two, in the middle, comes the (inclusive) disjunction. In this middle ( $p \vee q$ ),

the value-distribution of  $p$  will determine the truth-value configuration of the alternative (our sign  $\Rightarrow$ ); the value-distribution of  $q$  is given by the value-configuration of the conjunction (sign  $\Leftarrow$ ). In this way the definition of the middle value-configuration will be: in the variables of inclusive denial, i.e., in the value-distribution of  $p$  and  $q$ , *at least* one of the variables must be true.

$p \uparrow q$	$p \vee q$	$p \wedge q$
$t F \Rightarrow t$	$\Rightarrow F T T \Leftarrow$	$t \Leftarrow T t$
$t T \Rightarrow f$	$\Rightarrow T T F \Leftarrow$	$t \Leftarrow F f$
$f T \Rightarrow t$	$\Rightarrow T T F \Leftarrow$	$f \Leftarrow F t$
$f F \Rightarrow f$	$\Rightarrow F F F \Leftarrow$	$f \Leftarrow F f$

We may arrive at the middle value-configuration by another way too. Instead of the minimum, *at least*, we can use the maximum, *at most*, bringing the latter into relation not to true as previously, but to false: in the “inclusive or” *at most* one variable may be false. By this definition of the connective, the usual distribution of values between  $p$  and  $q$ :  $t-t-f-f$  and  $t-f-t-f$  gives the same result as the unusual, proving the validity of our unusual method. Another consequence is that the first column of the first matrix and the last matrix’s last column give us the necessary value-distribution of all the varieties of the connectives of (a) alternative, (b) inclusive denial, and (c) conjunction. A further feature that may be mentioned in this connection—although it is everywhere valid—is that the first and the last matrices are interchangeable, because there is no difference between the constant varieties’ value columns as regards their first or second positions in the order.

The main question now is which of the three synthesizing components is the deciding one. It must be negation. Our problem is therefore to give a concrete exposition of negation. We know already that inclusive disjunction with negated variables gives the same value-configuration as the disjunctive (misleadingly called “alternative”) denial. On the other hand, DeMorgan gives the following rule:  $\neg p \vee \neg q \equiv \neg(p \wedge q)$ ; the inclusive disjunction with negated variables is equivalent to the negated

conjunction (where the negation is directed at the connective itself and not at the variables). Tabulating these equivalencies of the indicated calculi and giving their value distribution and configuration, we may study these connections pictographically:

$p \mid q$	$\neg p \vee \neg q$	$\neg(p \wedge q)$
$t \ F \ t$	$f \ F \ f$	$t \ F \ t$
$t \ T \ f$	$f \ T \ t$	$t \ T \ f$
$f \ T \ t$	$t \ T \ f$	$f \ T \ t$
$f \ T \ f$	$t \ T \ t$	$f \ T \ f$

A claim that both kinds of denials equally well serve the reduction of the calculi still existing in mathematical logic may be explained by the fact that its sphere is that of abstract structural algebra, which puts on the same level things that in their mathematical immediacy do not seem relevant, and is not concerned with very important philosophical differences. It tries to get the maximum homogenization. There is no proof, however, that the deeper viewpoints connecting mathematics with philosophy would not be useful, even from purely mathematical points of view—though this does not coincide necessarily with operational effectiveness. I am convinced that on the borderland between philosophy and the particular sciences (*where the border is more connecting than separating*) the advantages are dominating factors, not inconveniences. Quine only reluctantly accepts this in the sphere of his logistical conception of the excluding “or.” Even in the fourth edition of his *Mathematical Logic*, Quine writes, “The exclusive use of ‘or’ is not frequent enough in technical developments to warrant a special name and symbol” (1961, 13). I do not know if we have a frequency distribution statistic about this question. I am, however, convinced of the erroneousness of this statement. The whole history of philosophy and the particular sciences bears witness to the opposite. False presupposition, or rather *présentiments* (taking the word in the twofold French meaning, both feeling and valuation), testify to the resentment against negation, negation not being originally a true-born child of mathematical thinking.<sup>6</sup> The resentment against negation emerged rather under the influence of the

philosophical hostility to negation as an indispensable category in the post-Hegelian, neo-Kantian era. The indifference to philosophy brings along with it the unconscious effects of some of the most problematic philosophical trends in other sciences. In the case of negation, it went to the ideal of a “negationless mathematics,” against which Brouwer fought an offensive for more than twenty years (1975a, 478). In such an atmosphere, the question about the origin of the “inclusive or” from diametrically opposed calculi could not come to the fore. A logical consequence of such a situation was a one-sided preference for alternative denial, on account of its dominant positive value-configuration.

***One or two bases of reduction to one “primitive”?***

Our analysis has already shown that there is a relation of simplicity and complexity between Sheffer’s two signs. As a way of considering this from the position we have already reached, let us compare their newly defined forms. Alternative denial ( $p \mid q$ ) holds true if at least one variable is false; this minimum contains in itself the maximum; it will be true with two false variables as well. Is it possible to express double negation ( $p \downarrow q$ ), defined by the complement of “at least,” by “at most”? Yes, but with certain mental reservations. The double (multifold) negation is true at most if both (or all) of its variables are false. This is an adequate definition, but in a stricter sense it is not absolutely exact, because there is no other possibility. To be perfectly exact we must say, it is true and only true if . . .

Comparison of the definitions of the two denials shows quite clearly that the minimum fundamental content is in double negation, even if we take into account the slight possibility of reflection of “as well” ( $\approx$  and), which is not unconditionally necessary. Even then, two conceptual moments stand only against the three.

If we accept the fundamental importance of these theoretical relations, it seems almost strange—in spite of its inherent mathematical correctness—to ask *whether Sheffer’s two denials may be used to express the calculus of negation*. The answer is yes. Negating the true state of affairs or assertions gives a false value;

negating the false gives a true value. And whereas the other calculi combine (at least) two statements, these use only one (even according to their uncritical protagonists), the alternative, which we take for the moment to be authentic. Enclosing  $p$  in parentheses and writing the value-configuration obtained from the negations of the  $t$  and  $f$  values of  $p$  under the negation sign, we form a matrix. Let us put it in the center, associating it with its equivalents, the two Sheffer connectives. On the left we have double denial, on the right alternative denial. All three are connected with the sign of equivalency. The tabulation is as follows:

$$\begin{array}{ccc}
 p \downarrow q & \equiv & \neg(p) & \equiv & p | q \\
 t F t & & f t & & t F t \\
 f T f & & T f & & f T f
 \end{array}$$

This result is a significant scientific discovery. Sheffer's achievement was brilliant because of its theoretical importance, even though technically—at least in the sphere of algebraic mathematical logic—it could not be used. Bertrand Russell, admitting the significance of Sheffer's stroke, did not accept the reduction of original connectives to this single symbol for the reason that it would have expanded the work—already voluminous (a work in three giant tomes!)—and consequently it “would have meant a very great labour” (1925, xiii).

But this is certainly not the eloquent testimony given by our tabulation, which tells more than even its discoverers imagined. Concentrated into an aphorism, its message is the same as the advice the “Hegelian” mathematical genius K. G. J. Jacobi gave on how to be successful in mathematics, “You must always invert!”<sup>7</sup> In our case, making this inversion means inverting the original question. Our way of putting the problem is whether we can express double denial by means of negation. On the one hand, it must be possible, because (leaving out the uninteresting third matrix)

$$\begin{array}{ccc}
 p \downarrow q & \equiv & \neg(p) \\
 t F t & & F t \\
 f T f & & T f
 \end{array}$$

and equivalency is a symmetrical inversive relation. The problem, however, remains because they must be *functionally equivalent*. One negation must have the same “reductive power” as two, which would seem to be impossible—impossible if we take abstract negation as only possible but imperatively necessary if we take the concrete negation as having specific differentiation in its autodynamics. The self-negation is a second negation, so we now have a twofold unity (the *Zweieinigkeit* of Brouwer), not one, or two, no singularity, or binary. In the philosophically deep strata of mathematical logic, every numerical attribute taken in isolation leads to mistakes. But if we want impetuously to give a characteristic, then the attribute singularity may be useful *in that deeper* sense of the mathematics of infinity, where it is not so much a quantitative as a qualitative determinant, a specificity. The specificity, i. e., singularity of negation, is its autodynamism. It posits itself in a double way. Negating itself, it negates itself in negating others. This stage is, however, only preparatory, the beginning of a synthesis of a higher order between negative and positive, without reaching the end. It is before and under the stage of what Lassalle to called “*prozessierende Identität*” [processing identity]. This *before and under the identity in process* grasps the mathematical logic tightly, fixing it numerically as *double* or *n-fold* negation, a negation according to the fundamental principle of the abstract identity of formal logic. Negation = negation. And what we repeat twice or more than twice need be said only once with the same “content” (where content is in fact only a material substrate, no less and no more, and not even the same, because this “identical content” means that it can be substituted at one’s discretion with anything having the same value). And conversely, what is said once may be said two times or many times. If  $avavav \dots \vee a \equiv a$ , then  $a \equiv avavav \dots \vee a$ .

The difficulty now seems to be the following. In this discretionary way, our “new” connective must be dependent on the state of “ontological” affairs, showing how many negations we must apply, and not on the internal structure of our logical

system. In spite of this appearance, it will depend on the latter, however, because the internal requirements determine the transportability of the other calculi into  $n$ -fold negation. The “ontological *state of affairs*,” even so, is a very small one, which, however, must not be bagatellized into a negligible quantity, because it is an adjustment to an empirical substrate, taken in its most abstract form of existence or nonexistence, regarding and denoting it, with some overstatement, as “truth-value.” The function of the determiner “some” is to remind us that the problematic of truth is much more complex than this theory presupposes; it is never and nowhere a question of isolated adequateness, but always that of a concrete relative totality. The truth-value in this sense must be relativized to mathematical logic as a relative abstract totality standing in concreteness under the requirements of philosophy as well as of the particular branches of science. It is best suited to information theory, computer science, and technology, where it attained its highly spectacular, world-shaping successes. Otherwise our assertion regarding an insufficient concreteness in deciding points may be extended to the important disciplines of mathematics, especially to research on its foundations.

Mathematical logic, as the finest specialists from Otto Hölder to Hao Wang more and more clearly acknowledged, is not a specific method of inquiry in mathematics, as Dieudonné believes, nor is its only and specific object mathematics; rather it applies mathematical, set-theoretical, and abstract algebraical methods to logical and even to ontological problems, as the so-called modal logic today shows. It would be a mistake to forget that in many ways dialectical thinking plays a crucial, although not necessarily conscious, part in it. Beginning with George Cantor, who in his own way understood something of dialectics (Szigeti 1971), through the anti-Hegelian Russell and the neutral Gödel, to the creator of “nonstandard analysis,” Abraham Robinson, they were compelled to think over their problematic dialectically, which they did without a conscious knowledge of dialectical theory.



*Dialectics and mathematics*

J. N. Findlay, the outstanding British “Hegelian” of the twentieth century (the quotation marks mean only that today nobody can be a rigorously “orthodox Hegelian”), expresses accurately what I have already roughly sketched out.

The fact, further, that Hegel uses superficially self-contradictory language in an illuminating manner, simply shows that Hegelian contradiction, like Hegelian identity, is not the nugatory, self-stultifying notion of the mathematical logician, but has an entirely different, valuable role. Hegelian dialectic has in fact a function complementary to the thought of *Principia Mathematica* and similar systems: it is the thought of the *interstices* between clear-cut notions, fixed axioms and rigorous deductive chains, the interstices where we are as yet unclear as to what our notions cover and what they do not, where we constantly stretch or retract them as we try them out on new material, where we are concerned to look at them from the outside, and see how well or how ill they do certain conceptual work, where we are concerned with innumerable loose, inexact, sliding shading relations of notions to one another which are not less important for being loose. Hegel’s dialectic corresponds to the sort of informal, non-formalizable passages of comment and discussion in a book like *Principia Mathematica*, rather than its systematic text, and it has the immense importance of that interstitial comment. The decisions and moves of thought that precede formal statement, and that lead us to discard one formal statement for another, are in fact the only part of a mathematicized system that has genuine philosophical interest, and all these are certainly dialectical in the Hegelian sense. (1963a, 218–19)

What Findlay does not say explicitly in his illuminating argument is only the following. All these dialectical decisions that seem to come absolutely externally into the particular subject,

are introduced into it, into its “systematic text,” mostly in an externalized (“alienated”) form; and so they appear before us in the guise of the special sphere, despite the fact that they remain discoverable and reproducible by means of deeper-going analysis. (Such was in miniature my reduction of the “inclusive or” to the dialectical poles of exclusive disjunction and conjunction.) It is certainly necessary that dialectical thinking, moving relatively in itself and for itself must be changed into its opposite, into a undialectical immanent movement of the subject. Further inquiry could point out those problem spheres of mathematics where the specifically mathematical and dialectical thinking “coincide” in their immanent or quasi-immanent realizations, or where and why the immediate or quasi-immediate presence of the dialectic disappears. The enumerated, by far not all essential, problems, show the complexity of the problem, as well as the possibility of the solutions.

Findlay made his own contribution even in regard to this problem sphere. In the essay just cited, he also declared the concrete possibility and reality of the intimate, internal unity between mathematical and dialectical forms of language: “Two of the greatest logico-mathematical discoveries of fairly recent times may be cited as excellent and beautiful examples of Hegelian dialectic: I refer to Cantor’s generation of transfinite numbers, and to Goedel’s theorem concerning undecidable sentences.” In his study “Goedelian Sentences: A Non-Numerical Approach,” Findlay again demonstrates the overt presence of dialectics (1963b, 65). A newer breakthrough to a quasi-open light of dialectic is the great achievement of Abraham Robinson’s *Non-Standard Analysis: Study in Logic and the Foundation of Mathematics* (1966)—the dialectical “metaphysics” of calculus, in the uniform of mathematical logic.

### ***Antidialecticism.***

It was Ryle’s inmost conviction that Hegel is not worth studying, not even for the sake of criticism. Often quoted in relation to dialectics is Peirce’s dictum: “metaphysics is the ape of mathematics.” This thought was condemned to ignominy by the

German mathematician Hahn even before it was formulated by Peirce. All these enmities created a lack of—to use Nietzsche’s word—“*intellektuelle Redlichkeit*” [intellectual uprightness]. In place of earnest discourses of serious problems came a kind of problem-shifting and philosophical arbitrariness. And this was not without sacrosanct historical traditions. The roots reach back to the failure of the revolutionary movement of the *German Vormärz* (period in German history between 1815 and the Revolution of 1848–Ed.). After 1848–49 “*ein Volk von Dichter und Denker*” [a people made up of poets and thinkers], as Gutzkow praised them (1836, 117), changed its course to become in the long run “*ein Volk von Richter und Henker*” [a people of judges and hangmen], as Karl Kraus was compelled to stigmatize them (1908). In the sixties of the nineteenth century, German academic life did not consider Hegelianism to be a venial sin; it became a deadly sin. The whole atmosphere of a gloomy and rigorous situation may be felt by reading the captivately tormented protest against it in the foreword to *Grundriss der Logik und Metaphysik* (1878) by Günther Thiele (the first philosopher to recognize the philosophical importance of George Cantor’s set theory, although Cantor was certainly not a prophet in his fatherland):

Must I ask forgiveness for my audacity for counting Fichte and Hegel to be classics of philosophy . . . ? Am I therefore standing in the way of Kant’s criticism or of empirical research? Kant’s transcendental deduction of categories of necessity presupposes accomplishment in individual details and, insofar as necessary, a knowledge of exact sciences by philosophers, this knowledge in itself being by no means philosophy. The task of philosophy begins only here: to find the *a priori* in the empirical data themselves; its task is to prove the presence of those categories that show their actions in construction of the world of phenomena. . . . With the conception of the task of philosophy in this sense, it goes without saying that one must study Fichte’s *Theory of Science* and Hegel’s *Logic* and if meanwhile I come across some ideas the truth of

which I must admit, then I am exactly compelled to stand for them regardless of whether or not they have a current course.

It is clear that the dominant tendency for more than a century—dominant but not exclusive—has been that there is neither intellectual ability, nor desire to examine and understand the whole of a range of problems in its *articulated totality*. Despite this, Thomas Aquinas—the initiator of the term *totality*—clearly understood the tremendous difference between abstract and concrete totality, between *totalitas homogenea* and *totalitas heterogenea* (*Sum. Theol.*, vol. 1, part 2, art. 2). The enemies of the concept of totality refer, on the one hand, to the sophistic assertion that a whole is incomprehensible precisely in its wholeness, and, on the other hand, to satisfy the demands of systems theory, complement the idea of incomprehensibility with a categorical statement: it is perfectly sufficient from a scientific point of view to fix and denote the abstract identity of a phenomenon with itself. Thus they are always able arbitrarily to pick up a “characteristic” mark from the object’s complex totality, with no regard whatsoever for the structural interdependence of the marks, quickly denoting this isolated form as primitive or primordial primitive, as happened—surely without dubious thought—with Sheffer’s two primitives.

***Hegel’s criticism of abstract identity and the scientific value and utility of the concrete dialectical identity principle***

From the viewpoint of “philosophical logic”—to use Croce’s memorable expression (his watchword was adopted by P. F. Strawson, as far as I know, without any reference to its origin and original dialectical content)—may we regard as authentically primordial any kind of sentential calculi with the exception of negation? It would be difficult. We have already seen, analyzing the conjunction ( $p \wedge q$ ) and the disjunction ( $r \vee s$ ), that they are complex formations. And this complexity, this involution is a characteristic trait of all symbolism. Every symbol must at the same time be identical with itself, and must differ from itself, as  $p$  differs from  $q$ , and  $r$  from  $s$  although they are all symbolic

signs, in this case variables for sentences. This relationship is seemingly purely external. But if we take into account the fact that every identity necessarily sets itself apart from others, the negation must be in itself. By this internal negation it negates itself in two directions. The essential direction is the way it constitutes itself to function as a symbol. It must have a complementary part, a "content," not this or that sentence with such and such a meaning, the latter being a material substrate and nothing more. The content is the scheme of the logical sentence in its generality, whether it is a variable or a constant. The second direction of self-negation is the possibility of expressing one and the same symbol by different signs. Their connection is conventionally regulated. In spite of this externalized component, in the whole formation, identity is the predominant power sustaining and reproducing itself in the given diversities. In view of the complexity of this and similar problems, it might be useful to return again to Hegel's synthetic identity.

Hegel was not yet hindered by the modern intricacies of formalization, which are perfectly justified and legitimized theoretically as well as practically by modern technical development, becoming the indispensable, essential means of progress, although not without engendering deeper social antagonisms in the given social establishment. The calculus of Ploucquet, the only one which Hegel knew, belongs to the shallow rills of the source-region of mathematical logic. It did not tax his intellectual capacity as much as the problematic of intensive infinity (with Cantor's nomenclature, *das Eigentlich oder Aktual Unendliche* [the authentic or actual infinity], which he solved, although, in contrast to Cantor, he was unable to differentiate internally between the different orders of actual infinity, not being a mathematician. For him the simple, fundamental facts of formal logic were at the same time ontological. The kernel law of formal logic, the principle of (abstract) identity (its complements, the principle of contradiction and that of the exclusion of a middle or third term, describe only the different aspect of the ruling first principle) declares the following "eternal" truth: Every phenomenon is

equal or identical to itself; it cannot be identical with something else.

The first point of Hegel's criticism is that if we take this principle in earnest, our thinking and speech would consequently be restricted to exclusive use of tautologies (namely, purely logical, not in the Aristotelian sense rhetorical). The logical subject *S* would and could only be repeated by the predicate *P*. As such a vacant generality, it is not false, but simply unproductive, without some kind step forward. The sense of this criticism is to prove the narrowness and rigidity of this principle, which does not reproduce even the factual structure of the mental process adequately, to say nothing of the identical processes of objective reality (self-reproduction at the same and on an amplified level). But we expect something else from a judgment, more than this vacuous repetition, as we are *de facto* use to doing in processing every thought. We are use to making a synthesis of an identical subject with a nonidentical predicate. In such a synthesis, the identity of subject takes into itself the nonidentical predicate, sustaining and enriching its own identity in the non-identical. It cannot not be a real identity without being an identity with something else, because its essential constituent, the ability to identify others through negation (which others may belong to its internal constituencies) will have disappeared. Instead of asserting, "A human being is a human being" or "A rose is a rose," we assert, "A human being is an animal [*zoon*]," "The rose is fragrant," or, on a more concrete level, in a genus-specific definitional judgment, "A human being is a *zoon politikon*" (Aristotle), or in modern times, "A human being is a tool-making animal" (Benjamin Franklin, Karl.Marx), "The rose is an ornamental garden flower." There are other possible definitional judgments for the same subject too. A human being may be a "*homo rationale*," the rose "a decorative sign of heraldry." Their truth depends partly on the sphere in which our thinking is moving, partly on the *genetically definitive* character of the judgment, fit to explain the varieties of the developed object. Following Hegel's usually unmentioned footsteps, formal

logicians began in vain to differentiate between *total* and *partial identity*, because they had not grasped the logical content of articulate totality.

I continue the elaboration of the significance of Hegel's principle. Whereas formal law one-sidedly emphasizes the constant in variable phenomena, Hegel accounts for both sides at the same time. And if one of the sides is dominant [*übergreifendes Moment*], its opposite, negation, is always demonstrably present. It is also clearly demonstrable that when one side takes the dominant role from the other, there must be a moment when both sides have equal force; they are equally preponderate, a situation that formal logic can easily change into a static stage (which does not correspond even to mechanics, to the principle of virtual displacement) bereft of conflict and struggle, concentrating it into a single and indivisible neutrality, or splitting it into two partial judgments, remembering only from dialectics tyrannically brought to an end that only one can be false, but that both can be true.

A further already epistemological-methodological result may be stated as follows. In the light of the Hegelian principle, we can always judge, not exactly, but in general, the distance which separates our conceptual mental picture from the intended extramental real object. It is similar to mathematics, where operating with specific symbols, we always know that our operation expresses the quantitative relations—external or internal or both—of certain things, independently of how many kindred relations, known or unknown, are expressible with the same mathematical apparatus and operations.

Another methodological consequence of the Hegelian principle is that a deeper and more comprehensive principle always allows the possibility of a narrower principle to be deduced from it, to move from the narrower to the deeper-lying and more comprehensive. The latter is a rather difficult bumpy road indeed, as we shall see when analyzing the problem of *modus ponens*. ("Easy" and "difficult," taken in their epistemological context, do not merely denote psychological categories, rather

they express different stages of complexity in the process of cognition, seen from the viewpoint of a collective or individual subject of knowledge.)

But let us now apply the just-mentioned principle of general validity to the categories of concrete and abstract identity. We must do nothing more than radically exclude the constituent not-identity from the principle of identity of identity and not-identity in such a way that not only their intimate connection with identity is negated, but even the possibility of their simple synchronicity, their coexistence in time, is negated also. This is achieved by the *principium contradictionis* (= *exclusion* of contradiction as the *Logique de Port-Royal* called it, better realizing the precision of *l'art de penser* than its successors). The exclusion is, however, not a starting point; it is already a result. It is a renouncement of the concrete negation, opposing abstract identity in the interest of creating a synthetic concrete identity. The renouncement as negation is by definition abstract negation, for the simple reason that it does not preserve anything from the negated, not even the memory of the way that led to itself, whereas negation can be itself until there is something to be negated. Abstract negation dissolves everything in complete annihilation. Or it would, if it could. For what remains as a result of abstract negation of the concrete identity? The identity (“not-identity” deleted) of identity. But the identity of identity is nothing other than the self-identity of identity. And as this “self” is being the selfsame identity, the abstract thesis simply says, identity is identity, identity = identity. But here the following—seen from the movement of the collective subject of knowledge—inevitable dialectical movement begins to work. The double emergence of identity points willy-nilly in the second identity to nonidentity, because the *second identity is at least quantitatively different* from the first. There is no quantity without quality. And this may go over from contradictory, i.e., from undeveloped difference into its fully developed contradictory, which is—not only according to Hegel, but DeMorgan as well—the contrariety, a qualitatively determined difference.



Whether something is a mental image or a material object, it cannot be identical with itself except by the mediation of something else, a hidden or open difference, as I shall prove in more detail. It must be still more evident—not to leave room for any doubt—that the most critical point of concrete identity, not-identity, is an undeniable moment of every act of thinking—not to mention of being—and as such has an intimate connection even with the principle of abstract identity. The plain fact that there can be no abstraction without concretion and vice versa makes it easier to get to real evidence in this profound problem. Let me use as atmospheric evidence the truth of the *inverted* paraphrase of Blake's famous epigram: "Do what you will, this world's a fiction / And is made up of contradiction."

Do what you will, this world's no fiction  
Though is made up of contradiction.

And the contradictory character of reality, including the forms of our process of thought, may be seen even when, to all appearances, any trace of contradiction must be perfectly excluded or annihilated. Let us therefore return to the question of logical tautology; perhaps there still remains something unilluminated.

Hegel did not pose the problem whether such tautologies as the judgment "A human being is a human being" could not have hidden in themselves the dialectical form of judgment, their opposite. Is their external form not merely externalized? Are they not merely expressions, the alienated, frozen forms of an internally elastic dialectical movement?

We have already seen that the same concept appears *twice* in a tautology, and the quantitative difference may signalize a qualitative one. But—and this is what we have not yet seen—in every tautological identity lurks a hidden qualitatively essential difference; it is *de facto* present already. Only the same concept—it may be expressed in synonyms—in its twofold existence veils over their constitutive qualitative differences that are necessary from the very beginning and that belong to their characters as components of judgment. The actual difference is an eminently

qualitative, functional one: *The selfsame concept is first the logical subject, and second the logical predicate, which are replaceable with each other.* The identity of meaning, however, suppresses the functional differences, the whole becomes a flesh-colored costume, covering the syntactical and semantical differences. But at important moments, the veil splits into its components, usually when the assertion emphatically puts another assertion out of the way. In such cases, it uncovers the real dialectical relations of the components, syntactically as well as semantically, i.e., the double unity, the form-content unity of the judgment. "A human being is a human being. Not a cannon fodder," says Bertolt Brecht. Here the tautology throws down its character. And this is already not merely a qualitatively functional, but an essential, relationship, the basis of the functional.

Such is the relation between the individual ("I") and the general or universal ("G" or "U"), valid to all tautologies and to all judgments, i.e., illuminating in them even the hidden contradiction, the mutual exclusion and implication of essential individual and universal. This internal relation unfolds itself in the structure of judgment, where the logical subject is *I*, and the predicate *G*. This is not as an extensional, quantitative relation—where the category of quantity should not be mistaken for the numeric character—not even a qualitative one. It transcends quality as an essential relation, excluding any "quantification of the predicate." (We do not forget the "anti-essentialist" tendencies of every kind of positivism whose proponents speak about "essentialism" as the worst of vices. The well-known boomerang effect is the result of this trend. Successfully relieving itself of any essential viewpoint in a number of important questions, their philosophy became unessential.)

In the sphere of essential relations, the universal does not simply subsume the individual (which, according to logical empiricism, belongs to the particular, because "some" equals at least one), placing the individual or particular under itself. It brings to light the essential constituents, or, as in the predicate of the last example, their totality of universal and individual or particular. By their interconnection, *I* and *U*, but not less so, *U* and

I, make the copula in the course of the further development of the judgment *S is P* more and more contentious, going hand in hand together with the birth of the presuppositions of theoretical thinking, and at a certain point turning into the strict formation of scientific theory. The primary fault, the *πρωτον ψευδος* [proton pseudos] of the most influential trend of formal logic was the abstract identification of individual, particular, and universal—which in dialectical logic I call *degrees of determination*—with their extensional (quantitative) relations, excluding both the qualitative and essential elements. Formal logicians murmur something about the primacy of quality to quantity. But actually this is only under constraint, under the pressure of superficial realities, because in this case, the quantity is always a quantity of something, i.e., of quality. They are compelled to bring back this “quality” through a window with the intention of correction, because in their primary intention they have already shown the door to it. This is useless. What they force back is already deformed. Instead of being the content of logical form, it is degraded to a rough material substrate without formal articulation. As the logicians of the Middle Ages—whom the moderns quote sometimes, without any profit—used to say, instead of being *materia designata* [designated matter], it is no more than *materia insignata* [undesignated matter]

An important conclusion of our analysis is the recognition that formal logic, from a systematic point of view, is a borderline case of dialectical logic. The latter cannot be conceived, as was customary in the Soviet Union, to be a general theory of the categories of dialectics, without undermining its true nature and significance. It must strictly be a theory of logical inferences with the component parts, with judgment and concept. There was, however, one great exception in the Soviet Union, the significant work of the outstanding Georgian philosopher S. B. Tsereteli *Dialekticheskaya Logika* [Dialectical Logic] (1971).<sup>8</sup> If one takes their relationship historically, however, formal logic has played a preparatory (propædeutic) role for dialectical logic, in any case already reflecting the most general dialectical principles. This propædeutic role of formal logic for the theory

of dialectical logic is almost exclusively *the* logical theory, by no means the immediate practical, collective, and individual forms of thinking and its everyday varieties. The latter forms, being nearer to real practical activity, are not homogeneous in the sense of formalism; they have important spontaneous-unconscious elements of dialectics. The complexity of this situation could be shown on the amphibolic Plato-Aristotle relationship, where the “materialist” empiricist Aristotle turned not only against the flight made by Plato in the heaven of pure ideas, but also against the dialectical high flying of Plato in the realm of dialectical categories and their internal connections, especially against his amazing achievement in the half-systematic–half-aphoristic dialogue, *Parmenides*. (Next to Hegel’s *Phenomenology* this is the most difficult and most profound work in world’s philosophical literature. It is analyzed in detail in my *Revolution of Scientific Thinking* (1984, 439–511).

The critical relation between Plato and Aristotle is the beginning of the repeatedly returning (by no means exclusive), topsy-turvy situation that becomes predominant in the history of pre-Marxian philosophy, in which, paradoxically enough, the ultimately metaphysically structured idealism carries forward the development of dialectics, whereas the *sui generis* dialectical character of materialism, manifested already at its birth in pre-Socratic philosophy, begins to commit itself more and more to metaphysical methods and views, not without producing important categories of dialectics, such as Spinoza’s *determinatio est negatio*. The final and definitive liquidation of this paradox was the world-historical act of Marx and Engels, connecting once and for all dialectics with materialism on a higher historical level in their materialist dialectics and dialectical materialism, conceived as an open scientific system.

Looking back to the analyses of our previous explanatory conception, someone not accustomed to dialectical forms and processes of thought may get the impression that the fundamental principle of dialectics and all that follows from it are rather difficult, *hypercomplex* relations. There is not a grain of truth in this. Or if there is, it does not mean more than the natural

requirement that every science must be studied with a serious effort of conceptual thinking [*Anstrengung des Begriffes*].

Let us take the concept of *hypercomplexity*—to think in clear-cut concepts—in a well-defined sense. “Complexity” is a manifoldly combined mechanistic chain of unanalyzable elements, where only the interconnection of elements by reason is intelligible, but not the substance of them. To compensate the deficiency of mechanicism, a vitalistic-organicistic conception is added to it, usually as a complement. “Hyper” in that sense is a life-unity, beyond the preceding structure, in its *élan vital*, which reacts to the complex mechanistic chain, as an external source of organic energy. *Élan vital* is unintelligible rationally only with irrational “intuition,” and intuition in this context cannot be resolved into logical categories. This adjustment of the organicistic view to the mechanistic is nothing else then expulsion of Satan by Lucifer, because irrationality and unintelligibility preserve itself on both sides.

In contrast to this, concrete identity is only “hypercomplex” in the sense of being adequate to the embarrassing intricacies of the real world. It is beyond, over, and above the bad extremities, eclectically and illusorically brought together. On the one hand, it stands above the “intelligibility” of mechanistic systems, above their abstract rationalism. On the other hand it may seemingly be grasped “under” the organicistic systems’ *élan vital* (according to Bergson’s assertion) by mystical intuitions and expressed exclusively in mental pictures and metaphors. Bergson does not know that there is another rational meaning of intuition not hostile to conceptualization if we take it simply as an abbreviated logical operation. *Dialectics*, as opposed to these ruling tendencies, finds in the dialectical contradiction the kernel of movement and vitality, and brings into its grasp the *élan vital* by recognizing its abstract, but real, constituencies in their internal interdependency. It is therefore only able to enlighten and make intelligible the intricacies of reality, being intelligible itself for itself as well, *when cast again on the level of today’s scientific development in its primordial materialistic form*. It was not the

fault of this incomparable mental microscope and telescope if the firms that produced the brand, after their initial successes—and these have been of world-historic significance—began to produce rubbish, with the result that the ground they have gained began to be lost, although not utterly irreversibly. We must return to the original method, in theory as well as in practice, because, used as a heuristic method, this at last is the method of *scientific discovery and practice*, which always urges the authentically active and recognizing subject to discern the facts of a progressing system formation everywhere, even in the seemingly most elementary phenomena unanalyzable by other methods, to grasp the different moments in the connections of coordination and subordination of the dynamics in process.

***The intelligibility and unintelligibility  
of fundamental logical principles***

After such an analysis, we may safely suppose that the principle of abstract identity in itself, without its relation to concrete identity, is *unintelligible*, and that, in contrast to it, concrete identity—one of the forms of contradiction—is *intelligible*. Both the less-trained mind of the average person on the street and the highly trained mind of the formal logician take precisely the inverse relation to be an unquestionable, axiomatic truth. To the average person on the street (not having the least notion of its abstractness) abstract identity is a hackneyed statement or truism, implanted by God in everybody's mind, as the height of rationality. Concrete identity, as a contradictory idea, seems like something vague and irritating, the very height of irrationality; the Devil take it. Is a human being an animal? Not at all! A human being is a human being, not an animal. This vulgar train of thought is the most frequent way of turning something from foot to head in topsy-turvy thinking. It is a paradigm of *everyday thinking*, but not of deeper, *immediate practical thinking*, in which the compulsory force of reality is much stronger.

We have already seen that the unintelligible moment in everyday thinking is precisely the point where it is seemingly the

strongest. But what is intelligible in the assertion that universal is universal? What is intelligible in repetition of the selfsame word, especially if one is not aware minimally that the selfsame word must be repeated twice to get even the *false appearance of really saying something*? And we know already that this quantitative difference brings our knowledge further to the recognition of the background structure of the judgment. Double-setting the same concept quantitatively (in Marx's term, *Doppeltsetzen*) leads us to the qualitative differences of the same word. And if some people are not even interested in the quantitative difference, why should they be interested in the hidden qualitative? It is true, in other, not logical, algebraic preconditions, that saying something once is the same as to saying it twice or thrice or  $n$ -times and vice versa. This is, however, not a genuine logical form, but the homogenizing tendency of the algebraic method of homogenization, bringing different things onto the same level for sake of operational uniformity in the sphere of syntax, which abstracts from semantic material. Add to this that, in truth-value calculi, the *principle of substitution is the ground of identity, whereas in logic identity is the ground for substitutionability*. No doubt the latter is correct in logic alone, because in the judgment there is a symmetrical substitutionability between  $S$  and  $P$ —as in tautologies—one side always must be the definiendum and the other the definiens. The difference between a simple assertion and a judgment follows with necessity from this situation. The latter must always contain, as its own threshold value, the determination degrees ( $I-P-U$ ); only by virtue of that do the grammatical subject and predicate become logical  $S-P$ , and not in the sense of quantity or extensionality as in the syntactical principle of substitutionability.

The notion ( $\approx$  *Vorstellung*  $\neq$  *Begriff*) of identity becomes somewhat intelligible when related at least externally to the different. A triangle = a triangle and not a quadrangle. This demarcation is at first glance external, showing at least the role of negation in the external definition. This negation being on itself, must be in and by itself. Its movement is not for others; it does

not aim only to limit anything else. The movement in and by itself is the determination of its own existence, by negating the attributes and independence of the elements that become its own substance as transformed materials. Three straight sections and three angles are not yet a triangle. Only by negation of the independence of these elements is it possible to synthesize them into a plane triangle. The internal movement is predominantly synthetic; the movement intended externally is predominantly analytic. I say “predominantly” only because the internal movement’s beginning is the negative transformation of elements, bereaving their independence and forming them into synthetic unity—second negation. And conversely, the external movement defining its own existence by demarcation makes for a better synthesis of the other beings, living things and nonliving ones, or the non-Euclidean triangle and the Euclidean, and vice versa, compelling us to recognize concretely the given specific spatial relations.

Intelligible and unintelligible identity, from the viewpoint of objectivity, is nothing other than abstract and concrete identity, respectively, the latter assimilating the contradiction. Both have abstract or concrete negativity for negative complements. But what is the basis in the subjectivity, in our consciousness, of this fatal *quid pro quo*? How do the intelligible and the unintelligible come to be inverted? I think the answer lies in a fact that is already coming into existence in immediate practical thinking, that abstract identity is unconsciously associated with present or not present, imaginative, perceptive materials, with notions (defined earlier as *Vorstellung*). This stealthy introduction of notions into the concept of identity makes abstract identity a rather blurred and variable mental entity, which is a more than problematic, unreliable substitute of the clear-cut differences existing in concrete identity. And the irrational moment of this kind of rationality comes to the fore with the “right-hand” (I use the term epistemologically) criticism of abstract rationalism, leading to a positive acceptance of irrationalism, which wishes to express the liveliness of life in metaphors and aesthetically



relevant pictures, as Bergson did (it goes without saying, in a perfectly illusory way), exemplary, but inevitably totally illusory. The authentically intelligible (concrete) identity is *inter alia* not intelligible because the sociohistorically determined practical and everyday consciousness, held captive in its partial and fragmentary experiences, is always reluctant to break through from obscurity into the light, to the world of illuminated natural things and social self, being the total and complete truth up to the present day and hour, something radically new and alien to them, compared to the customary half-truths and their frequent associations with half- and complete lies, proclaimed in every way, and trumpeted as loudly as possible, as the immanent parts of eternal truth. Plato was profoundly correct when, in the cave metaphor, he described the process of attaining adequate consciousness as that obtained by people who, when accustomed to be home in a world of half-obscurities, shadows, and darkness, are dazzled by the unknown, gloriously bright, and resplendent world of real truth. Contradictory forms and ways of thinking that had been unintelligible naivety of everyday consciousness become intelligible as in never-ceasing movement, being identity for those who are in the position of a breakthrough. From this time on, this process of truth becomes transparent, no obscurity or mystery adhering to it. And conversely, this sheds light on why the abstract identities of the vulgar, naive consciousness, such as differenceless identity with itself, as absolute self-identity, seem intelligible, purely for the reason that, in the semiobscurity of their notions, those visual and other perceptive elements are surreptitiously smuggled in, which, in sharp contradiction to their identity concept, realize a subconscious contradiction. Fetishistic, motionless identity, reified in and by itself, is in reality an alienated derivation of the identity moving in contradiction. To use the felicitous term of Lassalle, the *processing identity* or *identity-process*.

The processing identity, confirmed in mental movements, much stronger and undeniable, realizes itself in objective reality with poles excluding each other and, at the same time, implying that they form together a real, i.e., *articulated*, totality. This is

easily misrepresented by metaphysical thinking, which goes blindly past one of the two components, which is especially easy if one or the other side predominates in a given time interval, or if one is compelled to glance at the middle ground—the battle-field of “equally” powerful components. Then one tries to escape the spectacle of their intertwined struggle, as if the phenomenon would be a horrific Gorgon’s head turning every unfortunate spectator into stone. The way to make such an escape is the abstract neutralization without difference of the “middle.” How this mutual exclusion and inclusion realizes itself depends on the specific nature of the phenomenon, including those “external” conditions that are integral parts of its self-reproduction. If instead of these, other significant conditions emerge and come to the fore, then two possibilities are open. Either our phenomenon is strong enough to assimilate the new conditions with a partial change in its own nature, or it is not strong enough, and the consequence will be its destruction. In both cases the subject is a processing identity, be it at the stage of self-reproduction on the same level or in the process of amplification, either as a result of external or internal change, or of both.

The description of such processes by means of abstract identity is impossible for undialectical reason. It gives only snapshots of more or less arbitrarily snatched situations without their internal continuity. Each follows out of the other successively, with differing importance, and by this method even situations on the same level lose their internal dynamism. We must transcend this insufficient principle to comply with the materialist requirements, being faithful to the changing reality. The new fundamental principle goes back from the “empirical unity of oppositions” to its most comprehensive form, the *stem-form of every (absolute as well as relative) totality*. It is the identity of identity and not-identity, validating itself in the sphere of essence as well as of existence. Hegel explicates it in detail in the sphere of essence. In contrast to him, Marx materialistically uses it all round in the sphere of existence, where, in a much more complicated way, it realizes itself in the complex interactions of accident and necessity.

***Dialectic unity of contradictory  
and contrary oppositions***

Let us now pinpoint one of the most distinguishing features that plays a leading role in the standpoint of *antidialecticism*. This is the rigid, metaphysical exclusion of the relation between contradictory and contrariety. According to the spokespersons of this view, nonwhite is a contradictory (“merely”) negative and black a contrary (“merely”) positive concept, both in opposition to white. The thesis of antidialecticism is then: It is impossible to obtain [*herausklauben*] a positive concept from a merely negative one. They do not know or wish to know anything from the concrete negation, transforming white to black by negation through a series of gray. This example clearly shows how dubious is the third fundamental principle of formal logic, *the principle of the excluded middle or third term*. Arbitrarily or in special circumstances, you can exclude this third member, but usually you cannot, because there must be a middle between opposites belonging to the same kind, in this case colors, more specifically *opaques*.

Certainly, this kind of criticism of Hegelian dialectics is from the very beginning mistaken, as Josef Schmidt demonstrated in his excellent book, *Hegel's Wissenschaft der Logik und ihre Kritik durch Adolf Trendelenburg* (1977). We must know that up to now the whole of antidialecticism followed in the footsteps of Trendelenburg, at least bringing some modernity into the terminology. The significant logicians of the nineteenth century, in their historical nearness to Hegel, were still compelled partly to criticize some real theses of Hegel, but very superficially. Mostly they criticized their own fantastic imaginations about Hegel as a logician. In spite of the fact that the critics must have known the very thoroughgoing Hegelian metacriticism, especially that of Feuerbach (1969) and Rosenkranz (1834, 1–140) on Bachman (1833, 1–322; 1835), who was the originator of the *formal-logical antidialecticism*, they very characteristically followed the tactics of academic hush-up begun by Plato against the materialist Democritus. They did not perceive that the reason why Hegel

moved from the ontological unity of oppositions to the logical principles was not because he believed that logic is the universal key to everything, as even the right-wing Hegelians believed, and for which notion J. E. Erdmann coined the new generally accepted term *panlogism*. According to Hegel's authentic conception, being is not a logical, but a spiritual entity. And this spiritual entity may not be regarded as irrational, as his polemic proves against Schelling's obscurantist conception of identity, in which all cows at night are equally dark, or as the English proverb runs, where candles are away all cats are gray; it is dialectical. This spiritualism is his objective idealism, whereas dialectics already means "absolute idealism." But this absolute has in and by itself the moment of relativity; otherwise it could not be absolute, not being complete. As a consequence of the relative moment, his absolute is not turned into ice; it is in perpetual motion, essentially on the same level, sociohistorically on the ground of bourgeois civil society.

Rethinking and radically transforming Kant's idea of the relation between "*verstandsmässiges*" and "*vernunftmässiges*" *Denken*, i.e., rational and intellectual thinking, where Kant assigns a very shabby role to *Vernunft*, the role of a skeptical clip of the wings of truth, Hegel conceives dialectics as intellectual thinking, to be a higher form of rationality, i.e., "intellectualism." This *intellectual* thinking is for him in contradistinction to Kant's "*vernunftmässiges*" thinking. The diapason of Hegel's spiritualized being, as the spirit of being, extends from the perceptive visualization through the notion to the concept. It is for this reason that Hegel could not put the identity of identity and not-identity ontologically into first place. But it is pulsing without self-consciousness as a prime mover in the diversified area of being (even in the famous triad of pure being–nothing–becoming), as their internal law. The unconscious law arrives at its consciousness in the realm of the subjective spirit, i.e., in the severely logical part (theory of inference, judgment, concept) of Hegel's *Logic*. The sphere of being, as the field of perceptive visualization, is surely a very improper and inaccurate description. It is easy to allow the categories to descend from this

amphibious level of objectivity to the subjectivity of a Kantian transcendentalism, against which a good many arguments may be advanced, more awful than a barrage fire. Some of them already come from Hegel. Another possibility may be, as Trendelenburg realized, to make an “organicist,” half-Kantian, half-Aristotelian “correction” to Hegel when he tried to derive a movement that comes into existence in dialectics from oppositional poles, to derive it from the visual perception that was, according to him, common in existence and thinking, from a philosophically nonenduring notion (see Szigeti 1948, 92–94). Hegel was perfectly in line against such a misinterpretation, because at an early stage, he ingeniously anticipated the development of those branches of science that work with perceptually unsaturated concepts.

If a logical sphere, as an absolute autonomous entity, perfectly isolated from reality, could in any way have been brought into existence (e.g., constructed on an abstract value-system) then, as a consequence of the lost ontological contents, in the absence of relative totalities, then and only then would abstract identity play the preponderant role. But this would be a rule on nothing, incapable of determining itself and anything else. Even “pure” formal logic must have a material substrate, which is acknowledged in the thin assertion about the primary role of quality in relation to quantity. But a quality without essence, let me emphasize again, is not logical content, just poor material that cannot help in the formation of the unity of conceptual form and content. All that the conceptual form can do, and under certain conditions does do, is subsume the rough materials according to their extensions. This at best could be a preparatory stage toward essential determinants. This is, however, always a situation in which that which is different drops out of the circle of identity (even in the best case if the difference of *total* and *partial* identity is postulated). Thus there is no connecting link between identity as a general entity, and not-identity as a special entity, the latter being the negative different, and as such at least the germ of the positive different, the *differentia specifica*. *Genus proximum* and *differentia specifica* are here in completely

alienated spheres, notwithstanding the law that there is no general without the specific and vice versa. In the same way as there is no subordination without coordination, because the essential differences of the superficially understood coordinates necessarily demand the clarification of their axial relationships.

***The undeniability of the negation of negation  
and the dialectical crabwise movement***

Let us once again examine the assertion that it is impossible to obtain a positive concept from a negative one. Not-identity is such a negative concept; how can you dredge out of it something positive, in our case the concept of differentness? Well, not-identity is the negation of identity. But if you do not take negation as an immediate prompt annihilation of the negated—what we called abstract negation—but instead, take it as a process—concrete negation—then you will get the full evidence, recapitulated in the next chain of ideas.

Negation exists as long as there is something to negate. The complement of this thesis is: Identity exists as long as there is something to be negated, to be excluded from itself. The inversion of Spinoza's dictum, *determinatio est negatio*, necessarily follows if and only if we take the concrete and not the abstract negation, *negatio (concreta) est determinatio*. The duration of both movements depends not only on themselves but on the contradictory moments, without which they would be instantly annihilated, disappearing into nothingness. Thought, as a succession of intentional mental acts, would be an eternal recommencement. It would be completely dissolved before becoming anything. Its becoming anything would simply mean dissolution into nothing. In the eternal torment of an everlasting rebirth, the only mitigating circumstance would be the instantaneous passing away. The concrete process character of the second moment of Hegel's triadic formula, not-identity, explains why the sublated form preserves the identity in itself as its own moment, making out of the ("merely") contradictory a contrary moment, a positive negation, the difference. In the movement of its development, the difference begins to differ from itself, and before the fulfillment

of the higher synthetic identity, brings into existence a new transitory moment, differentness out of difference. This is the explanation why in certain conditions the triad is amplified into a tetrad, as Hegel, and his more gifted followers, such as Carl Ludwig Michelet, who, together with Ferdinand Lassalle (1919, 6:15), severely criticized the undialectical faulty reforms of Rosenkranz's *Wissenschaft der logischen Idee* (1858), knew very well. In the development of objectivity itself, this variable complexity of dialectical development manifests itself still more clearly.<sup>9</sup>

Our description was a faithful exposition of the negation of negation together with some implicit (*the survival of identity in nonidentity*) and more hidden (*the amplification of the triad into a tetrad*) phenomena. The fundamental form of this movement is undeniable, but not in the sense that there is no possibility of blocking its way with a barricade, as religious and secular existentialisms did. They stopped the process of contradiction at the bipolar structure of the opposition between identity and non-identity, substituting further progress with wriggling and tossing between the two poles, from Kierkegaard through Heidegger to Sartre, and in the meantime propagated the identity principle as the only possibility of an authentic existence, trying to approach it. But in vain. Not moving toward the ideal, they were localized in the same place. Kierkegaard once aptly called this the paradox of a "dialectical treading."

The negation of negation is undeniable in the sense that it finds a way everywhere. If there is a solution to a formally unsolvable problem, it leads further with its substantial search for the way out. Or if there is no solution, it denies the absurd (unsolvable imaginary) contradiction and sends us back to the starting point, either for a critical supervision of it, or—as in mathematical proofs which use the *reductio ad absurdum*—in the certainty that the original starting point *A* is valid and not its negative non-*A*. It is frequently unclear that this regression is, in fact, at the same time a progression in the total process of cognizance, because the original starting point is not quite the same as it was before our mental movements; we know much more about

it. For this reason this transient crabwise (i.e., sideways and backward) motion of dialectics—to borrow a term from musical harmonics—is a clear proof against Karl Popper’s wishy-washy talk that from a dialectical point of view it is not possible to understand self-reproduction on approximately the same level. In many ways this relates to the algebra of Boole.

The crabwise motion of sending us back to examine the starting point of a phenomenon clearly demonstrates, on the one hand, an important characteristic of dialectical development, the specificity of dialectical return. On the other hand, it directs our attention to the importance of the—always relative—starting point. The starting point is conditioned by internal and external factors. In studying the development of a phenomenon, we may ignore the external factors if they have no considerable effect on our object. If they do, then they belong somehow to the critical factor. Let us study the problem of the starting point in the case of Boole.

### ***Boolean algebra, polyvalent logic, and dialectics***

Boole’s starting point was the theory of equations of the second order. He did not consider as easy the seemingly “trivial” marginal case where the square of the base is equal to the base. However, the simplest usable equivalent equation is  $x = x^2$ . After the first step in solving the equation,  $(x^2 - x) = 0$ , the replacement of the equation with the equivalent one,  $x \cdot (x - 1) = 0$ , gives us the “logically” utilizable roots,  $x = 1$  and  $x = 0$ . These may be easily identified with the values true (1) and false (0), where true and false mean the most abstractly conceived state of affairs, in which there is or is not something. This fact could bring us to problems of epistemology, because logic, especially formal logic, and, to a certain extent, dialectical logic, study the correct operations with logical forms; dialectical logic, at its higher level, serves as a guide leading to the relative totalities of concrete truth. Be that as it may, Boole’s algebra, as an application of mathematical methods to logic, with its correspondence between algebraic forms and the factual state of things, brought a new and fundamental science into existence. Boole’s algebra



(*The Mathematical Analysis of Logic Being an Essay Towards a Calculus of Deductive Reasoning*, first published in 1847) has been developed further—Frege’s ingenious contribution is noteworthy here—proving its theoretical and practical significance for all of modern science.

Its sphere is not so much the treatment of factual truth, “*vérités de fait*,” as those connections that may be brought together in hypothetico-deductive systems, the “*vérités de raison*.” If Boole would have known the achievements of classical German philosophy in dialectical thinking, he could have arrived at the idea that there is still an obvious mathematical way of transcending the rigid bivalent opposition of true and false. This would have been achieved by going one step further in the analogy to the mathematics of the cubic equation,  $x^3 = x$ . Replacing  $x^3$  by  $x \cdot x^2$  and rearranging, we then seek the solution of  $x \cdot (x^2 - 1) = 0$ . And  $(x^2 - 1)$  being equal to  $(x - 1) \cdot (x + 1)$ , the transformed formula can be written

$$x \cdot (x - 1) \cdot (x + 1) = 0.$$

We have now three roots instead of two. Let (+1) be true and (−1) false; the third root (0) may easily represent a neutral value. The neutral value is the result of a neutralization process between true and false having on itself the traces of its birth, being “neither true nor false, and hence . . . a third logical value” (Łukasiewicz 1970, 87). Philosophically there are of course different possibilities of interpretation, from the dialectical to the agnostic. This would have been the opening way to polyvalent logic, because the intervals between the given three values could have been divided into transfinite continua.

History waited for more than three quarters of a century before a mathematician took that step. It is another question that the founder of polyvalent logic, Jan Łukasiewicz went on a different path for taking other values (first in *Ruch filozoficzny* 5 [1920], 170–71). The introduction of the three-valued model begins the working out of polyvalent logic. “Poly” means in this sphere more than two, at least three, and not even only three. For

intervals between the two extreme values (+1) and (−1) and the “neutral” third (0), or, following Łukasiewicz’s mathematically more adequate designation (free from the problem of imaginary numbers),  $1-\frac{1}{2}-0$ , these intervals may be apportioned to the infinite series of rational numbers, associating with every number a “truth-value” that expresses the significance of the values according to their intention and extension.

The trivalent triad is nearer to dialectics than bivalent dualism, in the sense that it is a more elastic instrument for the formalization of *some aspects of the dialectic* than the bivalent, by means of which Georg Klaus, absolutely misunderstanding the nature of dialectics as well as that of bivalent logic, tried to give a formalized scheme of the identity of identity and not-identity (1959, 52, 100, 166). Fortunately, much better experiments have been made, not accidentally with the help of polyvalent formalism such as in Kosok 1966, Sabelli 1977, Asenjo 1965, Gauthier 1967, and Rogowski 1967. A critical analysis of these studies, especially Rogowski’s (unfortunately, there is no English translation of this interesting work), which is the most consistent attempt to show that “the logical sense (*sic!*) of Hegel’s conception,” would dispel illusions that formalization can give the key to a full understanding of Hegel or Marx. On the ground of materialist dialectics, the real character, the whys and wherefores of mathematical and mathematical-form formalization, will become clear. That is why dialectics, in its scientific form, is ultimately not formalizable, not even if a formalizer acknowledges that in polyvalent logic immanent dialectical elements are the *metalogic* of the bivalent and not the other way round, the bivalent of the polyvalent. The latter is today the generally accepted standpoint, in spite of the fact that quantity, as external and indifferent determination, which attains its reality content by means of so many mediations that it becomes applicable to everything and—without radical transformation in the direction of concretization—to nothing. Or as Bertrand Russell put it in his skeptical way, almost on the verge of mathematical nihilism, at least from the standpoint of applied mathematics,

“Mathematics may be defined as the subject in which we never know what we are talking about, nor whether what we are saying is true.” (1937, 75). The title of the essay in which the quote may be found, “Mathematics and the Metaphysicians,” is an early paper, written in 1901, but Russell maintains this assertion to the last, no matter how his philosophy changed in other ways. The ruling externalized form makes it impossible to accept mathematics as the primordial form of thinking, as a mode of the form-content unity that always existed on the horizon of philosophical expectations and, somewhat altered, in the different branches of the specialized sciences as well.

We must not forget, meanwhile, that according to our earlier thesis, mathematical or symbolical logic is not a philosophical discipline, but a mathematical one, being an area of modern abstract or structural algebra. This is why its denotation as mathematical logic is much more correct than symbolic logic, not for the reason that its main study would be mathematical connections and configurations, but as a result of its fundamental position of applying mathematical methods to the form of thought, as Boole expressed it very early on, to “find calculus to deductive reasoning.” And in the higher spheres of mathematics the conscious and unconscious ways of dialectical reasoning not only latently validate themselves in deeply hidden implications, but in developed forms as well, in open forms existing in and by themselves. When and where this breakthrough comes does not fall within the scope of this essay. It is enough to assert in this context that it is most frequently in higher mathematics, which does not mean that we assume a gap between elementary and higher mathematics, after Cantor and Dedekind, with their set-theoretical approach carried out the bridge-building between the two domains that formerly, in the everyday view of mathematicians, had been regarded as entirely separate. Experiment with formalization of certain aspects of dialectics may be quite useful if it is conducted with an awareness of its own limits; otherwise, overgeneralizing brings with it the loss of dialectical effectiveness and elasticity. On the other hand, breakthroughs in the specialized branches of science partly

justify the elasticity of dialectics and may partly help its further development.

***Dialectical transitions in the formalized sphere***

As an example and proof of my last assertion about the dialectical significance of the breakthrough in the particular sciences, I shall turn again to the false assertion about the impossibility of transition from contradictory to contrary concepts. Its historical origin goes back to the anti-Platonism of Aristotle, one of his few, but noteworthy, mistakes concerning the changes in the fortune of dialectics. Here Aristotle's justified standpoint against the extremist (consciously *antimaterialist*) idealism of Plato and (to use the language of the great philosophical debate between nominalism and concept-realism of the Middle Ages) against the principle of "*universalia ante res*" passes over into a critique of the dialectic, because in his empiricism (*universalia in rebus*), which has some of the distinguishing features of materialism, negative concepts and judgments serve only for the removal of false notions without any objective movement and change. This mistake (productive as a precondition of his logical formalism, which he calls *analytics*) in many ways stands in contradiction to his deeper dialectical efforts, but it is in any case an important mistake. On this ground I find highly problematic the unrestricted validity of Engels's dictum claiming Aristotle as the most outstanding dialectician of antiquity after having examined closely all forms of Aristotle's dialectics. (Lenin had a good reason to mention once, only once, in his *Philosophical Notebooks* (1972, 258) the Stagirite antidialectics of Aristotle.)

After Aristotle's "critical opinion," the tearing asunder of the dialectical unity of contradictory and contrary became more or less accepted, but in the nineteenth century, even those logicians who came to logical problems through mathematics—independently of Hegel—began to scrutinize skeptically this position. The outstanding exponent of this approach was the English mathematician Augustus DeMorgan. In his *Formal Logic or the Calculus of Inference* (1847), published the same year as Boole's first book, DeMorgan closely and critically examined this metaphysical separation, and came to the important conclusion that

contradictory and contrary judgments and concepts may be definitely separated if and only if we operate according to the following rule: In a contradictory relation between two events or judgments, one, but only one, must be necessarily true, the other being false. A tossed coin must be either head or tail. A color is either white or not white. We may accept this on the basis of DeMorgan's idea *if* the negation is an abstract one and *if* there is no middle between two possibilities. But this case is very rare. Even a coin must have an edge, which eventually may play a part. Is there a third, a middle case, between the given white and the not-given color, not white? Yes. What is negated must, first, also be a color in any case, the universe of discourse being color; it cannot therefore be a pencil. But the universe of discourse, secondly, may be smaller, more concrete. The colors may be either *bright* colors, brilliant as garden flowers (and even here, complementary colors) or *opaques*, as in the typical example of white and its contrary black. But the way to get from white to black is the concrete negation of white: white–nonwhite–black, or, if black is the original point of departure, we go black–nonblack–white. And the critical mind of DeMorgan is well aware of this aspect of the problem. Taking as examples a number of sentences and words from standard and scientific language, he came to the following conclusion, “A *contrary, therefore, is a complete and total contradictory*” (1847, 5), a discovered discovery, since Hegel had realized this almost half a century before. This does not affect DeMorgan's originality. More than probably, he had not heard anything about Hegel before the British Hegelians appeared on the scene.

Where DeMorgan was not sufficiently critical, for want of a conscious dialectic, is in the problem of the middle point, the neutralization of the opposites. Is this a transition in the *total development of the negation* (and this is what he tells us in the assertion we quoted)? Then there must be a point where the negated white is no longer white, but not yet black. For our example with typical *opaque colors*, it must be gray. (Let us exclude from our explanation the physical ground of this

physico-physiological process.) But there are many different shades of gray. At the beginning, with the negation of white, it is a very pale gray. From this to the middle of the scale between white and black, the opaque color grows grayer and grayer until it reaches a midgray. This we take to be grayness without further qualification. Compared to the second end of the scale, extending from grayness to black, where the color gets closer and closer to black, the gray of the first part is faint and pale, the white color is still preponderant, whereas in the second line, being dark and deep, the black becomes the preponderant quality.

The methodological weakness of DeMorgan is his strong analytical attachment to the linguistic expression of thought in the sanctified traditions of English philosophy. Despite his half-French sounding name, there is hardly any element of Cartesianism in his spirit. He does not examine the important logical consequences pointing to any trend other than formal logic, but immediately continues the already cited assertion in the immanent trend of formal logic (although he does add something new): “and a little consideration will make it appear that the decisive distinction between contraries and contradictories lies in this, that contraries may be both false, but of contradictories, one must be true and the other false” (5). It does not cross his intellectual horizon—just as the possibility of trivalency instead of bivalency did not cross that of Boole—that the whole problem leads us radically into another direction, through the internal relations of judgments, instead of the formal-logical relations, represented graphically by the “logical square,” to their dialectically logical relations, represented graphically by a “logical triangle.” The latter today is not just a logical possibility; one of the ways of developing it was realized in the pioneer work of S. B. Tsereteli referred to earlier. (The explanation of the problem belongs to a systematic exposition of dialectical logic, which is not within the frame of this discussion.) All these reflections are valid under two suppositions: 1) that the materials of concept and judgment are different from their content, whereby which of the two is preponderant depends entirely on

the sphere (formal or dialectical) in which we are moving; 2) that there is a dialectical transition between concept and judgment; concept is the result of fulfilled judgment; judgment is an unfolded concept.

DeMorgan's new definition of the difference between contradictory and contrary in the framework of formal logic became productive for the development of mathematical logic thanks to the mediation of Stanley Jevons. He began the quantification of the sentential calculi along a new line, in which DeMorgan's contradictory negation, in its most developed form, become positive negation, the contrariety.

The positive and negative universals (in symbols,  $\forall$  and  $\neg\forall$ ) and the positive and negative particulars (in symbols:  $\exists$  and  $\neg\exists$ , which contain the individual (*some* being at least one), connected to each other by substitution, will be dynamic. As Quine aptly says in the *Methods of Logic* (1950, 86), "Universal and existential quantification *are intimately connected in meaning through negation*" (italics added). The positive quantifiers turn into the negative, the particulars into universals, and inversely. The process, as such, of turning into the other disappears (it is carried out in the activity of the cognitive subject), only the end result being before our eyes in its rigid equivalency, notwithstanding the fact that the eggshells of its origin are clearly before our intellectual comprehension in the form of the *meaning of negation* (i.e., not only syntactically but semantically) mentioned by Quine.

With the concept of negation, the ontological side of the problem also appears. For the sake of simplicity, we may ignore some subtle problems that would necessitate a deeper analysis of our dialectical-materialist ontological conception. What we disclose may, I hope, be sufficient in this context, and will be strengthened further by our later reflections. The symbol  $\exists$  does not denote simply the particular, it also indicates that something, an  $x$  exists ( $\exists x$ ), or does not exist ( $\neg\exists x$ ). For this reason, it is called an existential quantifier. This  $x$  may be practically everything: an object, a situation, a state of affairs [*Sachverhalt*], an idea, etc. Let  $x$  be a horse: =  $Hx$ . It must have some attributes, one by which we designate it: a horse is quadruped (" $Q$ "). And

since all horses are quadruped, we put the sentential calculus in universal form:  $(\forall x)(Hx \rightarrow Qx)$ , which means, following the symbols strictly: *Every  $x$ , if  $x$  is a horse, must therefore be quadruped*. Is there a way of expressing the assertion in another equivalent sentential calculus? Yes, there is, through negation, not forgetting DeMorgan's statement about the identity and difference between contradictory and contrary opposition. From the universal we return therefore to existential quantification. The equivalent formula is:  $(\neg \exists x)(Hx \wedge \neg Qx)$ , there is *no*  $x$  (not one) such that  $x$  is a horse *and not* quadruped. This connection between negated particular and asserted universal is symmetrical by the prime-moving negation. Not only is the universal existential calculus expressible by the negated existential, but the posited existential by the negated universal. The color of our horse is now chestnut ("C"). The existence of a chestnut horse in positive existential quantifier is  $(\exists x)(Hx \wedge Cx)$ , there is at least one  $x$  that is a horse and chestnut colored. Its equivalent is  $\neg(\forall x)(Hx \rightarrow \neg Cx)$ , it is not always the situation that if  $x$  is a horse then  $x$  must be chestnut colored. It is an interesting point that the connection between the two sentences in the universal quantified sentential calculus is expressed with implication, in the existential always with a conjunction, or any other equivalent operator.

From this and from other considerations it is concluded, almost without exception, that there is a deep chasm in this difference between existential and general calculi. On one side, real existence, one or some individuals appear; on the other, merely ideational existence—*notions*, concepts, universals do not exist outside of the mind. Those who hold this view do not perceive that by such a notion the universals are conceived unconsciously as individual existences, because, in this case, only in their imagination could they have obtained real existence outside of the mind. But this is impossible on the ground that they do not show themselves separately from the individuals. According to them, therefore, they have only a subjective existence or validity. On the contrary, however, universals have a real objective existence, which validates itself within individual existence, as their common structure, in an infinity of individuals (natural laws), in



some individuals (the living world), and even in one unique individual (each newborn historical phenomenon). There is no universal without the individual, but the opposite is true as well, there is no individual without the general. Under such circumstances, there is no reason to interpret the transitions explained above with a quasi-dialectical abnegation, in defense of an obsolescent nominalism. Nominalism turned from its onetime materialistic empiricism into an extreme subjective idealism a good many decades ago. The only explanation for such a misconceived interpretation is a prejudice toward this nominalism.

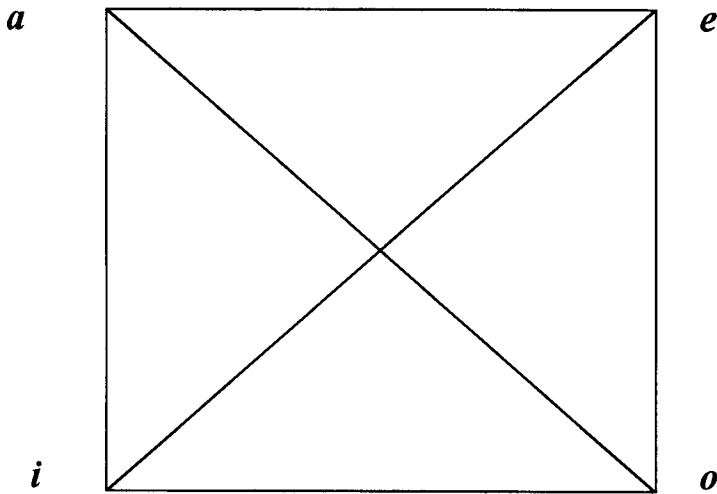
In the formalization of formal logic, unconsciously emerging dialectical (“musical”) roulades and scale-passages, taken all in all, do not mean that the formal character of the formalized variety has been completely broken through. It remains in the metaphysical rigidity of the framework. In the final analysis, it preserves nondialectical rigidity, whereas internally, as a consequence of algebraic synthetic methods (algebra from Arabic: *al* [the] + *jabr* [reunify the broken parts]), the elasticity grows rapidly and, together with it, the extensive comprehensibility and applicability.

To sum up, I should like to claim the following. The equivalencies treated here, as the outcome of a dialectical thinking process, demonstrate their dialectical character in the significant role of negation, the transformation of the negative into the positive (and vice versa). They bring their ultimately undialectical character to the fore in the fact that, in the definition of positive and negative particulars and universals, the extension remains the ruling determiner in their algebraic transformations as well, in a certain sense not only to the power of 1, but also to the *n*th power.

### ***The problem of the limits of extension***

Formal logic signals its quantitatively and qualitatively determined judgments by using the following abbreviations of Latin words: “*a*” = *asserit*, universal positive; “*e*” = *negat*, universal negative; “*i*” = *asserit*, particular positive; “*o*” = *nego*, particular negative. In the case of universal positive and negative

judgments,  $a-e$ , only one can be true but both may be false. Their relationship is a contrary, diametrical opposition (it is raining everywhere today—it is raining nowhere today). Positive and negative particular judgments,  $i-o$  are in a subcontrary relation. Only one can be false, but both may be true (in some parts of the country it was raining today—in some parts of the country it was not raining today). Subalternating is the relation of universal and particular positive and negative judgments:  $a-i$  and  $e-o$ . If the universal is true, the particular is true as well. But not the other way round. The universal may or may not be true, independently of the truth of the particular. More important are the contradictory relations:  $a-o$  (all swans are white—not all swans are white) and  $e-i$  (there are no irrational numbers—some numbers are irrational). One must be true, the other false, both cannot be true or false at the same time. A geometrical square allows us to give a graphical picture of all these relations.



The extensional or quantitative ( $\neq$  numerical) relationships between judgments expressed in the quantifiers of mathematical logic level up the essential relationships of individuality–specificity–generality, just as the particular swallows up the individual. This is one of the important internal limits of every formalistic conception, which becomes immediately evident when we take into account what we said about the first appearance of a historical phenomenon, be it natural, social, or ideational. As Darwin maintained (not to mention the more adequate explanations of Marx), varieties, under suitable conditions, are the beginnings of new species, and species are the developed results of new varieties; these problems and solutions must also be reflected in a Marxist theory of logic. This is one clear shortcut on the limits of formalism.

But there is another, too, growing out of the quantitative extensive nature of formalism and formalization. The judgments and inferences given in the logical square may, in their algebraic equivalencies, be regarded as a small part of the circle of equivalencies that play an important part in the algorithms of inferences in modern algebra. (The so-called logical theory of relations is not part of any kind of logic; it is a purely algebraic theory, and as such, one of the areas of abstract algebra.) At first glance, a considerable number of these equivalencies reveal themselves as shorthand ways of writing a syllogism, an immediate syllogism, a syllogism with one premise (the other waiting for adjunction) called *enthymeme*. But there are others, supposedly neither immediate nor mediate, not even resolvable into such. Since DeMorgan's remark that all of Aristotle's logic could not prove that "because a horse is an animal, the head of a horse is the head of an animal," some modern logicians again and again give hand-waving proofs merely with examples like this that they cannot be translated into inference. Let the readers convince themselves by inspection whether such assertions, called *literature of paradoxes* are true. Against unproven conviction, I give my own conviction without proof: DeMorgan's assertion is wrong. There are several ways to solve it, immediate as well as mediate, formal as well as dialectical. The general problem,

however, remains, that the solution of some examples is not yet the general solution. We shall take some steps in a direction that is the possible way of getting from some empirical problems to the level of a general solution.

### ***The modus ponens and recursive inference***

To the much utilized forms of judgment belongs the *modus ponens*, which plays an important part in mathematical logic. If  $p$  is given, then also  $q$ ;  $p$  is given, hence  $q$ . According to the arguments of formal logic, we have the antecedent together with the consequent (in formalized form merely a logical consecution, *implication* or *conditional*). The character of the first judgment is hypothetical (conditional); it does not tell us anything about the existence of the antecedent. The second judgment, succeeding the conditional, must tell something about it. If it rains, then the earth is wet; it is raining, hence the earth is wet. But—and this is important—we cannot validly infer the antecedent from the consequent. If it rains, then the earth is wet; the earth is wet, hence it rained. The usual argument against this is that the consequent may result from different causes, the earth's wetness may be caused by a water-truck or by a flood. The truth matrix of the conditional sentential calculus, transformed to the inference described, gives the same result. The *if . . . then* conditional or implication is always true except if the antecedent is true and the consequent false. The argument—with or without the authoritatively frequently cited Aristotle—is at least problematic: out of a true antecedent with correct application of logical principles it is inconceivable to get a false consequent. The argument turns against itself: why is it not inconceivable to obtain a true consequent from a false antecedent with correct application of logical principles, at least without negation, and without room for negation according to the presupposition? If any implication beginning with a false antecedent necessarily must be true, whether its consequent is false or true, then compared to other connectives this would be the strongest with a consequent that is from the viewpoint of truth-value perfectly indifferent. Of course, there is *no objection against this generalization if we do*

not take it from a logical point of view (because it has a very limited relation to the twin categories ground-consequence as a relationship of being and logical knowledge), and if we accept it as an algebraic device that works within the restricted framework of its consistently construed deductive-hypothetical system.

We shall put the *modus ponens* into the symbol language of algebra. Then to the conditional, as the first premise, comes the second by means of the conjunction. The point, being one of the many marks of parentheses, signifies the end of the premises. After the point, the conclusion is connected with the premises by the implication sign. We give the algebraic formula in non-quantified form, quantification being in this connection unimportant.

$$p \rightarrow q \wedge p. \rightarrow q$$

A nonquantified (“open”) formula, just as a quantified (“closed”) one, is universally valid if by every permitted distribution of truth-values ( $t$  and  $f$ ) it becomes true, or, what is the same, not falsifiable. Let us try to falsify our *modus ponens*. The first member (to the parenthesis point) is the antecedent of the implied consequent, which is the last member. The conditional is false if the antecedent is true, the consequent false. The conjunction is always false except if both members are true. When we replace the variables with truth-values in the permitted way, the process and its result will be:

$$\begin{array}{lll} t \rightarrow f \wedge t. \rightarrow f & \text{and} & f \rightarrow f \wedge f. \rightarrow f \\ f \wedge t. \rightarrow f & \text{and} & t \wedge f. \rightarrow f \\ f \rightarrow f \equiv T & \text{and} & f \rightarrow f \equiv T \end{array}$$

The *modus ponens*, as the failure of the falsification experiment proves, has a general validity. The concluding implication twice showed a false antecedent, and this is a certain stigma of truth. We would have obtained the same result if we had quantified our formula by putting it into a closed form.

More important for us is the question whether the *modus ponens* also preserves in a formalized form the specificity of the

original if we put the consequent, instead of the antecedent of the first conditional, in the place after the sign of conjunction, and the antecedent in the last place of the inference, after the second implication sign. Does the *modus ponens* lose or not lose its general validity? It will be once false, once true, occasionally changing its value. This is therefore a mixed result, which is called *consistent, i.e., noncontradictory*, and which cannot mean once and for all contradictory; it is not a (negative) tautology, not similar to its counterpart, which was once and for all true as a (positive) tautology. Replacing the truth-values once again in the formalized formula realizing the inference, we get:

$$\begin{array}{lcl}
 & p \rightarrow q \wedge q. \rightarrow p & \\
 f \rightarrow t \wedge t. \rightarrow f & \text{and} & f \rightarrow f \wedge f. \rightarrow f \\
 t \wedge t. \rightarrow f & \text{and} & t \wedge f. \rightarrow f \\
 \text{Now } t \wedge t \equiv t & \text{and} & t \wedge f \equiv f \\
 \text{hence } t \rightarrow f \equiv F & \text{and} & f \rightarrow f \equiv T
 \end{array}$$

The permittedly different truth-value distribution gives in the first column *F*(alse), in the second *T*(ruth), and this verifies the formal-logical conception.

In spite of that, it would be rather difficult to acquiesce in such an obviously one-sided character of the process of cognition. The amphibious trait of recursive inference would make shaky, extremely shaky, the way from reality to theory, or even to the immediate practical and everyday consciousness. What remains would be exclusively the praxis of trial and error, although our starting point in those areas is most frequently the consequent and not the antecedent, at least when new problems emerge in the practical and theoretical horizon, not internalized yet into the theoretical consciousness.

There are different forms of *modus ponens* (*modus ponendo ponens, tollendo ponens, ponendo tollens, tollendo tollens*), but it is enough to take the most simple form. It is not expedient to change the original dependence on the antecedent-consequent relationship, although there is a reaction—which has at least a modifying power—from the part of the consequent changing two

pole positions. The antecedent as a ground in ultimate analysis remains the stronger category, the consequent as a ground the weaker, although, in general, finer, more differentiated. And this relation validates itself even in their interactions and changes of place. If we exchange the originally “monistic” ground with the “pluralistic” consequent, then the new pluralistic ground cannot answer for a monistic consequent becoming a ground, except if it will be the beginning of a transformation into a new and higher relative totality in which the new ground begins to imprint its pluralism on the monistic ground transforming it. But then we return principally again to the primary relationship between the stronger antecedent and the weaker consequent, as a new starting point for dialectical development.

Some logicians do not accept *modus ponens* as inference, because there is no middle term in it that could enter into a relationship with the extreme terms and mediate between them.

The *modus ponens*, however, being in its proposition major a hypothetical inference, and in its proposition minor and conclusion, categorical, may be transformed from a “mixed” form into a purely categorical (syllogistic) inference. And this transformation clearly shows the existence of a middle term. If it rains, the earth is wet—written in categorical form: *Each time it rains* the earth gets wet. The categorical (minor) proposition and the conclusion, freed from their seemingly subjectless character and put in accurate form give: *Here and now* it is raining. Therefore: *Here and now* the earth is wet. All elements of a correct formal-logical syllogism are together.

*Each time it rains* the earth gets wet.

*Here and now* it is raining.

---

Here and now the earth is wet.

There are the extreme concepts. Term minor: the *here and now* of space and time. Term major: *the wet earth*. Term medial: the *(falling) rain*.

This last term is in perfect agreement with the rule of the *distribution of terms*. The rule means, to put it simply, that a

term is distributed if in the premises it appears at least once in its complete extension, universally, and not particularly, i.e., undistributed. In the major proposition, the middle term is distributed (“*Each* rain . . .”); in the minor, the middle is undistributed, but it is enough to understand its general character, because it is in the context of inference and not in an isolated minor proposition.

Every concept has quality (positive, negative) and quantity (universal, particular). Previously, we took together quantity and quality by means of the memorable word-abbreviations (*a-e*, *i-o*; see pages 322–23). Combining this with the structure of judgment, *S*(subject) *is* *P*(predicate), in such a way that the copula (“is”) is replaced by a proper abbreviation, we may tell which term is distributed or undistributed in the different judgments. In the universal assertion, *S a P*, *S* is distributed and *P* undistributed as above in the major proposition of the categorical inference: *Each* rain makes the earth wet. In the universal negation, both *S* and *P* are distributed, because negation mutually excludes their extension from each other, as if we would have said: This wetness is not caused by rain. The particular assertion *S i P* cannot have distributed terms. Both *S* and *P* are undistributed. This is why there is no (conclusive) inference from merely particular premises. In particular negation, *S o P*, *S* is undistributed, and *P*, being negated, is distributed: In some southern parts of the country did not rain. We may get the usual picture of the syllogism if the middle term is written, as “*M*” in place of “*S*” or “*P*” in the premises—in the conclusion it is already dismissed, there being no place for it—according to its mediating role. To give a picture of the usual scheme of syllogisms, we write the original *modus ponens* transformed into its categorical framework; this will be the first Aristotelian scheme, according to the “topological” position of the middle term. The formal structure is preceded by the example used above.

Any rain makes the earth wet.	<i>M a P</i>
Here and now the rain is falling.	<i>S i M</i> ( <i>Scheme I</i> )
Here and now the earth is wet.	<i>S i P</i> ( <i>of Aristotle</i> )



***The crisis of the extensional determinant and the identity process representing degrees of determination***

In the previous scheme, the middle term of the first premise played the part of *S*, and in the second, that of *P*. This is, however, not necessarily so. It can take in both premises at once, either the part of the subject or that of the predicate. Add to this fact that not only the originally given middle term may function in a syllogism as the middle, each of the extreme terms, the minor term and the major term, may—and in a system of syllogisms *must*—function as the middle term. A self-evident corollary to this is that the original middle must take over the previous part of extremes. These considerations may show that the middle is not so much a middle term as a mediating term, mediating between the extremes. Such circumstances, along with a good many others, led to a situation where the logicians began to feel the insufficiencies of the prevailing extensional conception, including the lack of distribution of predicate and particulars, and went back to rethink and experiment mentally. Out of this, two tendencies came into existence, more or less mutually exclusive. One appeared in the 1840s, in the wake of Hamilton and DeMorgan, after G. (≠J.) Bentham's somewhat earlier initialization. Its characteristic is the complete extension of quantification, DeMorgan's contribution implying the qualitative aspect of developing the negative into a full positive, the contradictory into a contrary, the latter being a completely and fully developed contradictory (1847, 92). DeMorgan's experiment was the really productive, pioneer work. The other trial was made in the 1860s by Stanley Jevons, professor of economics, logic, and rhetoric at Manchester and London, with his experiment in *Pure Logic* (1864), "pure" in the sense of its subtitle, *The Logic of Quality Apart from Quantity*. The conception is built on principles showing that *qualitative relations* are not merely a noisy wind:

Reasoning in quality and quantity, in intent or extent of meaning, being considered apart, it seems obvious that the comparison of things in quality, with respect to all their points of sameness and differences gives the primary and

most general system of reasoning. It even seems likely that such a system must comprehend all possible and conceivable kinds of reasoning, since it treats of any and every way in which things may be the same or different. *All reasoning is probably founded on the laws of sameness and difference* (italics added) which form the basis of the following system.—My present task . . . is to show that *all and more than all the ordinary processes of logic may be combined in a system founded of comparison of quality only, without the reference to logical quantity*” (italics in the original). (1890, 4)

The quantification is a subsequent task. This step was done also by Jevons in 1874 in his *Principles of Science* (1907), but the finished form, based on matrix-system quantification, was done much later (along the fundamental lines already indicated).

The orientation on empirical (even more and more merely linguistic) facts was transcended and a healthy lesson was drawn from the mathematical transcriptions of philosophical problems. There was no tendency that would have been able to question whether there is a way to conceive a *Wesensschau* [display of essence] not in the Husserlian sense, not a looking at, and at the same time constituting, subjective essence-entities in our mental acts, but by every available method of cognition mentally grasping and conceptually expressing things, relations, situations in their universal essence. If somebody thinks that the universal essence submerged in the individual phenomenon thereby becomes in itself individual, then he or she is forgetting that there is no individual without a universal. The concept of essence in Marxist philosophy is strictly and effectively demarcated from the concept-idealism of Plato, from the ideas hovering before and above real material things (who knows in what cloud-cuckoo-castle-land?), as well as from the concept-realism of Aristotle, which does not externalize, alienating the ideas and universals from the space-time continuum but puts them directly into things, relations, and situations, as their interior essence, as their moving force as soul, the movement

coming from their internal impetus, a longing for something. Externalized or internalized, the question is not a topological one. Idea remains in both forms idea, something in their nature different from material things, which cannot come into existence as a result of material interactions, whereas the opposite is possible. Material things are created out of the idea in this or that way, as a result of conscious or unconscious teleological acts, which are at the same time theological acts.

The logicians who were making pioneer efforts in several respects, and who, in their rigid nominalistic “anti-essentialism,” were against dialectics, rejected dialectics without thinking when they looked steadily in the eyes of something like dialectics, as the Bradley-Russell debate demonstrates (Bradley 1914, 276–310, Russell 1903, 41, 43n, 47, 90, 99, 161n, 221, 224, 228, 248, 471). The materialist dialectic was unknown. If it had been known, the situation would have been the same as with Jevons and the labor theory of value of classical political economy, which he, without much ado, excluded from the realm of science. When he spoke about theoretical questions of economy in a logical relation, he always falsely identified exchange value with use value.

The key to the solution lies in the ontological categories of individual (*I*), specific (*S*), and universal (*U*). In the adequate reflection of these objective categories in the branches of theoretical philosophy: ontology, logic, and epistemology. Let me reiterate: *I*, *S*, *U* (*G*) are primarily not quantitative, not even in the Jevonsian sense qualitative, but essential categories constituting the universal, general nature of individual phenomena. Their primary character, therefore, cannot be that out of a set of things, or sentences, etc.; they denote one or some or all the individua. In a given totality of phenomena, *I*, *S*, *U* (*G*) constitute the essential nature of phenomena in their production and reproduction on different levels of concretion, in their possible ramifications for different spheres, genera, species. The situation is the same in subjective, spiritual development. And for the simple reason that all these processes do not happen in a material being’s transcending apotheosis world, not even in an immanent world moved by

spiritual energies (Bergson), where it is useless to imagine these energies as a god in things and situations, because its transcendent character does not follow from its topological spacing, but from the spiritualization of the moving force. Degrees of determination must of necessity be material relationships between material things and situations, or what is just the same in a single material thing, between its different moments, constituting the totality of material tissues. This totality of tissues does not exist without individuality, and no individuality exists without universality, as both extremes do not exist without a mediating specificity. And this means the world in its wholeness, i.e., in its totality. But there is a difference between the *whole world* and the *world-whole*. The first, the whole world, is the subject of different specialized branches of science, examining the articulated world in its specific and individual details. And this subject is therefore the articulated world as an *individual totality*. Because if there is no totality without universality, or universality without individuality, then there cannot be an articulated world without the interactive poles of *U* and *I* that creates the specificity *S*.

The second, the world-whole, is the subject of philosophy. It is the world articulation, i.e., the *total individuality*, the subject of philosophy, for the same reason as the preceding connection, only with the significant difference that this second objective connection, as well as its scientific mirror, is dominated by the universal, just as the first was dominated by the individual. The two different determinations realize their objective unity in the world process, subjectively in the progressive interaction and interpenetration of the specialized and nonspecialized sciences.

*The objective dialectical synthesis of the twofold unity in the historical world process gives the only uniquely total, and not representatively, relatively existing complete and compact individuality.* The sensible phenomena—Marx in conscious contrast to Feuerbach called them *sensible nonsensible*—given in and around us are but an infinitely small part of the universe about which we know something through its manifestation in them. Terminologically, "I should like to define matter in this context (without a claim to final solution of this deep and far-reaching

problem, which must be explicated in another study in its full richness) as objective entity consisting of *corpora* (*Körper* as Marx and Engels called them), having two attributes in dialectical interaction: *thing and relation realized in their immanent space-time unity*.

They are corpuses (*corpora*, bodies) independently of their character, whether they are predominantly things such as a mountain, a flower, or a Grecian urn, where the relations mediate the things, or they are predominantly relations such as a gravitational or electromagnetic field, or social relations of production, where the things mediate the relations. The bodies, as unities of things and relations bound in space and time, are individua (not to be confused with the social category of individualities), in spite of the philosophical misinterpretations of Heisenberg's uncertainty relationship, or in spite of the fact that two gold atoms are perfectly the same—if one can speak about a perfect sameness in the sphere of microentities. We could not speak about their sameness if they did not have the minimum definition of individua, a space-time separation from each other, in consequence of which there are many more of them than two, although never enough, as Aeneas says in Virgil, speaking about the *auri sacra fames*, meaning the homicidal hunt for gold.

The identity of identity and not-identity, as the innermost law of material processes, is valid also in the categorical connections of *I–S–U*. *Reism*, a fashionable philosophical trend in Poland that dualistically cuts the world into things and relations, makes a mistake not only in that in its idealism—surpassing that of Thomas Aquinas—it is unwilling to proceed to the concept of the corpus, but also it conceives both elements as eternal ready-made entities, taking the relation to be just another thing. And the reason why the world is not homogeneous, why it is heterogeneous, as ultimately every idealism with a religious character imagines in the apotheosizing belief of divine homogeneity, because alienated from its creator it is split in two halves, is because the innermost negation of matter divides the world not only qualitatively but also essentially into things and relations.

The thing as identity bears the internal and external negations in itself as an internal form and as an external difference from others. The negation of negation brings the concrete body into existence with its synthetic identity. It remains in this state until a new radical change effects its transition into another body. The identity, full of quantitative, qualitative, and essential changes, the last of which may be its passing away, not only individually but generically, verifying Lassalle's term, is processing identity or identity process. It tangibly shows itself in the empirical phenomena of the Heraclitian "strife and battle." The fundamental identity process, as its internal law, makes its way through complications, external and internal deformations, breaking its way through empirical accidental occurrences and countertendencies. For example, new societal formations clearly marked by many strengths and advantages did not emerge and develop in the twentieth century without serious deformations and even setbacks. Ontologically such a process always begins with the emergence of a new phenomenon, notwithstanding that the new emerges under the existing conditions of preceding bodies, be they natural, social, or ideational. (Marx more than once used the concept of *Gesellschaftskörper* for the characterization of a given society).

Epistemologically, the process of cognition begins with the body's *analytic* approximation, with revealing things mediated by their relations or relations mediated by the proper things. The *synthetic* work begins as a result of this stage, our conceptual scheme now helps the mental reproduction of the whole course of the examined process in its objective historical character. It is extremely important to know that the two stages of the process of cognition effectively always use the opposite movement: the analytic uses the synthetic, and the synthetic the analytic. This is one of the main guiding principles of epistemology.

***Analysis and synthesis of the categorical degree of specificity***

If one takes all these presuppositions into account, it must be clear that the dialectics of *I*, *S*, *U* is shaped by the identity of

identity and not-identity. It is what it is on the basis of its internal and external negation, of the re-creating and demarcating movements. We complemented the extremely fruitful idea of Spinoza (*determinatio est negatio*) with its inverse (*negatio concreta est determinatio*) in the conviction that the great master of philosophy would approve of this. As a re-creative movement, as a posited negation, *I* must be non-*I*, i.e., *U*, attaining a higher degree of determination.

But what is the situation with *S*? We wanted to get *S*, and instead of *S* we got *U*. What can we do to reach our target? Let us try now to clear up the matter, simplifying it somewhat in order to stay within the limits of the *usual demonstrations* (permissible for me if it is afterward possible to eliminate the moment of simplification). Let us consider the matter in the following steps.

Non-*I* represents the “first” degree of *U*, which we called *S*. Why *S*? Because there is a higher *U*, let us denote it by the uppercase *U'*. Or, if you prefer, there are still higher and higher *U*'s: *U''*, *U'''*, . . . , practically an infinity of them. But these do not interest us now for the reason that we want to grasp the self-same object, and if we grasp our object in its relative totality, then the operation we are to realize repeats itself again and again, on higher and higher levels, which in the case of object-determined necessity may be lower and lower levels as well. We must therefore continue our operation where we left it, with our higher *U'*. Downward from the viewpoint of this higher *U'* must be *I*, just as *U* had to be upward from the lower level. As a consequence of this, the provisory *S* is double posited. In the middle of two inequalities is the position of *S*, but from two different approximations. It is not a compact unity, rather it is a twofold unity, from down upward and from up downward. Is there a possibility of a dialectical synthesis (in abstract algebra the usual symbol is “ $\otimes$ ”)? It is possible if we cut *S* into two halves: *S'* and *S''*. This dividing of *S* has its basis in its difference. But we may give closer determiners of the two marked *S*'s, taking into account their constituents, expressing them by the lower case “*u*” and “*i*” as sublated capitals, *U* and *I*. Then  $S' = I < U = u \otimes i$ ,

where the presence of the lower-case “*u*” in first position expresses *u*’s predominance over *i*. We express this dominance by the Latin word *commune*.  $S'' = U' > U = i \otimes u$ , called *proprium*. With *S*’ having been explained, the formula for  $S''$  goes without saying. The lower-case letter in *S*’, the dominant *u*, becomes *subdominant* in  $S''$ , and the earlier *subdominant* *i* is here dominant. This analysis makes possible the synthesis. As a matter of fact  $S' \neq S''$ . But  $S = S' \otimes S''$ . Now the right side is commutative:

$$S'' \otimes S' = S' \otimes S''$$

Both sides are equal to *S*. To visualize the complete synthesis graphically, let us place the rows under each other as a column, writing the sign of synthesis in the middle between them:

$$\begin{array}{c} S = (S' \otimes S'') \\ \otimes \\ (S'' \otimes S') = S \end{array}$$

For better understanding by means of complete visualization, let us put the values of the two marked *S*’s into the formula, noting that which of the two rows must be the upper depends on the concrete content of the object examined:

$$\begin{array}{l} S' = \{(u \otimes i) \otimes (i \otimes u)\} = S'' \\ S = [\otimes] = S \\ S'' = \{(i \otimes u) \otimes (u \otimes i)\} = S' \end{array}$$

Anybody may justly point out that this verification is rather arbitrary because we took a second higher *U*, i.e., *U*’, out of our bag; nobody knows how, or who, could have put *U*’ into it. The usual, seemingly sufficient answer to such questions is that the existence of an infinite number of higher *U*’s is a self-evident truth, and it is even proved in other branches of science—in our present case in axiomatic or nonaxiomatic set theory. For everyday purposes, this is certainly acceptable, at least it is accepted. But if we take seriously Leibniz’s principle of sufficient reason, nobody can assert that this argumentation fulfills the norm prescribed by the words quoted. I accept therefore that it was a



subsidiary hypothesis that was not necessary, introduced only for sake of facility.

The observant reader could be forgiven for thinking, after all we have said and tried to prove about the autodynamism of dialectics, that there is no need for  $U'$  as an auxiliary hypothesis, since negation becomes necessarily reflexive. What happens if we go further with the process of negation, negating our first  $U'$ ? The second negation,  $U'$ , not being a retraction, cannot annihilate the first  $U$ , and return to  $I$ .  $U'$  negating  $U$  not only relativizes it, but creates an indispensable, essential moment for its own growth as well. This is because in the process of its own development from the differences in  $S$ , or, to use the proper and clear-cut concept of the mathematician E. Huhn, from the “*negative identity of difference*,” it may sweep through into the positive identity of difference to the highest level.

This must be a *sublation* (the excellent translation of *Aufhebung*, a culminating term of dialectics, by the first English Hegelian, Stirling), negating and conserving the two preceding degrees of determination,  $i$  and  $u$  as the content of the synthetic  $U'$  on a higher, richer level ( $U' > U$ ). This is the whole accomplished process. But in this higher  $U'$ , in the *positive identity of difference*, the hidden moment of negativity may be the first step of a higher (or lower) level. This problem leads us back to some of the ontological conceptions that I explained in my *Revolution of Scientific Thinking*, where in opposition to the epistemological method followed here ( $I-S-U$  called the “lower determinant”), we took  $U-S-I$  (called the “upper determinant”) as the starting point, because it is the fundamental ontological determination (1984, 205–22, 238–59). By lower and higher determinants we denote only the initial member of the series, whether the series begins with  $I$  (lower) or with  $U$  (upper), according to its degree of determination.

### ***Hegelian mathematicians and idealistic dialectics: A historical interlude***

E. Huhn was the assistant of the great C. G. Jacobi, under the latter's professorship in Königsberg. Jacobi, the genius of

mathematics, the pioneer of the inquiry into elliptical functions, a relatively young victim of tuberculosis, in this time a fearful enemy of great mathematicians, was one of the small number of mathematicians, who, in terms of their philosophy, now completely forgotten, were declared Hegelians, although as critical spirits they also knew something of the weaknesses of Hegel's philosophy of mathematics. In the 1820s, Jacobi, in the years of his first professorship in Berlin, where Hegel had many admirers from the most distinguished circles, starting with Altenstein, the minister of culture, continuing with top-ranking officers who went to listen to his lectures, down to the wide groups of university students, must have known Hegel personally. His dictum, his advice on how to attain significant new results in mathematics: "You must always invert!" is undeniably imbued with the spirit of dialectics, which is all the more remarkable, because Gauss, the *princeps mathematicorum*, and the great majority along with him, including the "Herbartian" Riemann, endorsed Kantian philosophy. Even Dirksen, Hegel's fellow professor, an expert in higher analysis, who belonged to Hegel's circle of friends, being a disciple of Gauss under the impulse of his master, was philosophically a Kantian.

Jacobi's assistant professor, Huhn—who died still younger of tuberculosis, before his professor—explained the why and wherefore of this Hegelianism, in his case, ardently, turning openly and sharply against Kant. Only Georg Cantor criticized and dismissed Kant more openly and markedly, allying him with Pyrrhonian skepticism, because of the conceptions of antinomy and approximation that he viewed as discrediting to human intellect (Szigeti 1971, 289). Huhn wrote a long essay on dialectics and mathematics in 1844. After him, almost ten years later, a substantial book (of two hundred pages) by a Hegelian mathematician, Hermann Schwarz, was published (not the same as Hermann Amandus Schwarz, a well-known expert in conformal mapping). Schwarz's book *Versuch einer Philosophie der Mathematik verbunden mit einer Kritik der Aufstellungen Hegel's über den Zweck und die Natur der höheren Analysis* [An Essay on the Philosophy of Mathematics connected with a Criticism of

the Positions of Hegel on the Ends and Nature of Higher Analysis] may be viewed as the swan-song of the Hegelian mathematicians. Jacobi and Huhn were dead and Hegel was regarded as a dead dog. This was the brief upsurge and rapid decline of one of the promising scientific tendencies of the German *Vormärz*.

The introduction to Huhn's comprehensive study, "Über die Möglichkeit einer philosophischer Behandlung der Mathematik" [On the Possibility of a Philosophical Treatment of Mathematics] (1844) gives a very good description of the superiority of the Hegelian dialectical philosophy of mathematics over Kant's conception. Huhn supports his assertion amply and ably in the dialectical handling of concrete mathematical questions. He is convinced of the complete erroneousness of the mathematization of philosophy, especially by following the paradigm of the Euclidean deductive-axiomatic system, even in the case of Spinoza, not to mention the lesser gods, Wolf, Ploucquet, and so forth. Today, when philosophy finds itself once again compelled to fight a battle for autonomy, all that Huhn says has a contemporary relevance, although the standpoints against which he fought had a more differentiated and refined appearance, not to mention their important and practicable parts:

Is there any possibility for a philosophical treatment of mathematics? When there was nobody to treat philosophy itself philosophically, this possibility was out of the question. Philosophers insisted rather that mathematics is the only science that is can be treated philosophically, looking at its systematic structure with greedy eyes and borrowing the method from it. The *Elements of Euclid* and other so-called systems of mathematics provided a pattern for the method. . . . Hume, the skeptic, held every knowledge uncertain and accidental, except mathematical knowledge. No one had the slightest idea about the philosophical, the authentic scientific method. Kant himself put together arbitrarily his philosophical theses, which do not win further determination from themselves at all, and not from their internal movements; their logic has nothing to do

with the content of philosophy, with metaphysical concepts; the triplicity of the dialectical process, according to which the concept posits itself as its own other, and comes first in this other to sameness with itself; he has no understanding of the negative identity of difference; in his antinomies he could not conceive the possibility that one idea and at the same time its opposite should be conceived; he did not grasp the unity of unity and its opposite, although it is precisely this that constitutes the speculation of philosophy; the things-in-themselves must not be contradictory; he did not allow himself to conceive of knowledge as a cognition of the things-in-themselves; the subject that is able to make concepts was, in his eyes, too feeble for that, the *ought* [das Sollen] of *approximation* was the highest point of solution of the contradictions of intellect [*Vernunft*].—It was Hegel who developed and demonstrated that consciousness of the inner form of the content of self-movement is the only true method to expound the philosophy. Every moment determines itself dialectically by itself to a higher degree, reaches back into the lower, and integrates its content. The content of thinking is the universal and necessary being, since the nature of the object, and the object is the thinking itself. (no. 21: 161)

If the object itself is not necessarily thinking, since Huhn considers accepting objective idealism together with dialectics, he takes one step, not a very resolute one, toward ridding himself of idealism, neither wanting the Euclidean system to apply to philosophy, nor its inverse, to make philosophy from mathematics. He puts the question thus: How does the universally valid dialectical method function in the sphere of a special—notwithstanding formally universal—science? How far does it transform and recreate the methods of mathematical proof and verification, giving them a complexly unified [*unitas multiplex*] methodology that is in agreement with its roots in general philosophy? Nowadays such questions are usually dismissed, and not only the

questions, but the published works that deal with them too, as happened over half a century with the interesting book by Gaston Casanova: *Mathématique et matérialisme dialectique* (1947). Only in the last decades have things begun to move, not so much on systematic, as on historical grounds (e.g., *Hegels Philosophie der Natur* [Hegel 1968]). Huhn cannot be suspected of having a modern partisanship, which is why it is very instructive to see how he demonstrates the validation of a conscious dialectical thinking in the light of the mathematical inquiry of his age, even as regards elliptical functions.

Huhn was not optimistic that his conception would be easily and quickly realized. He was aware that some of the greats felt its necessity, especially the French. He liked and cited the maxim of Laplace: “Préférez les méthodes générales, attachez-vous á présenter de la manière la plus simple, et vous verrez en même temps qu’elles sont presque toujours les plus faciles.” [Prefer general methods, spare no effort to expound them the simplest way possible, and you will see at the same time that they are almost always the easiest.] And he also quotes the aperçu of Chasles: “L’esprit de la géométrie moderne est d’élever toujours les vérités, soit anciennes soit nouvelles, à la plus grande généralité qu’il se puisse” [The spirit of modern geometry is always to lift truths, be they ancient or new, to the highest generality possible] (Huhn 1844, no. 25:197). He did not know the interesting voluminous book by his own compatriot, Constantin Frantz, who, before his Schellingian “internal enlightenment,” his *Kehre* [turn] to a very modern Heideggerian style, was an orthodox Hegelian. Franz’s book, *Die Philosophie der Mathematik. Zugleich ein Beitrag zur Logik und naturphilosophie* [The Philosophy of Mathematics, Being also a Contribution to Logic and Natural Philosophy] (1842), was published two years before Huhn’s work. Therefore, Huhn wrote in his first article, “Until now nobody has experimented with a dialectical treatment of mathematics” (1844, no. 21:162). This shows clearly the lack of organization of “Hegelings” in this field. (Frantz, from a mathematician and hopeful Hegelian, soon

became a civil servant and publicist of the Prussian ministry of foreign affairs, from where he was dismissed in the revolutionary days of 1848.) But Huhn knew his own countrymen, and he realized that there were some people who felt the problems, but they lacked the energy to do anything.

They did not keep any account at all of Hegel and his philosophy. . . . According to what has already been said, the mediation of theses with each other, their demonstration, is by no means a subjective act without objectivity. It is the object moving further by itself, imbuing itself with content. In this way the mathematical inference is not merely formal, because it essentially belongs to the object. Hegel appreciated mathematics as it exists today, and not as it ought to be and may one day become. Although the possibility and necessity of a philosophical treatment of mathematics already exists, there is no hope that such treatment will be realized in the near future. It would require tremendous labor. The jumble of old theses has become, so to speak, second nature to mathematicians, and it would be exceedingly difficult to grasp and sustain everywhere the many a compounded number and space configurations in their logical connections. (no. 25: 197–98)

***“Negative identity of difference” (E. Huhn)  
and the substantial middle term of inference***

This historical interlude was by no means gratuitous. It was intended to reflect a practically unknown period of the history of the relationship between dialectics and mathematics, in which the Hegelian period, despite its brevity, had a fruitful beginning and an aftermath, as I have attempted to prove (Szigeti 1971), that reached its acme in George Cantor’s wonderful mathematical production. And this production was not without previous experiments, which Cantor very likely knew but did not want to reveal, knowing that a charge of Hegelianism would bring him

into a more difficult situation than he had had in Germany, not being a prophet in his own country. What he said in later years about his philosophical background is sometimes almost as telling a tale as his lectures positing Bacon as the author of Shakespeare's dramas, where he was unaware even of the simple fact that the rich descendants of Lord Bacon propagated this calumnious falsehood through mercenary hack-writers.

But let us now return to our systematic task, which is, of course, not a systematic exposition, but merely a sketchy outline of dialectical logic and its real interaction with formal and formalized algorithmicized logic. Our next step is to follow, using the sharp-minded Huhn's category of the neoliberal identity of difference, the subtle problems of *recursive inference*.

The relation between *I*, *S*, and *U* cannot be the negative identity of difference without objective and subjective self-movement. If our concept were filled with sense material, even then this sense material alone would not determine their self-movement, as must be evident already from the necessity of the unity of material and categorical form. Their autodynamism is relative and finite. They exhaust in their proceeding all the possibilities of internal development, returning for new experimental material. Neither the origin, nor the fulfillment, has a merely contemplative character; it is much more active and practical. A third moment unifying contemplation and activity is the Greek *poiésis*, *poiéma*. This has the unambiguous sense of an object-setting activity, resulting in the realization of the idea, whether the object is a utensil, a tool, or a poetical work. In this way, logic must not be pure logic in the sense that it needs exclusively to discover the laws of scientific theoretical thinking. Logic is in some way always and ever present in the human mind's activity, in immediate practical thinking, as well as in artistic creation and scientific production, in all instances with special traits.

Grammar is not the same as logic. But logic, beyond doubt, determines some important points in the loom of language. The theoretical logic of dialectical materialism must take into account all the varieties, shades, and transitions from one form to the

other. As regards practical everyday thinking, independently of the circumstance how and in what proportions it is mixed up eclectically with formal and dialectical elements, we can take it for granted that, under the pressure and compulsion of objective reality, this common form of thinking is inclined to connect more or less dialectically the distinct elements of reality that formal-logical thinking separates severely—if necessary quite correctly—and often treats as common defects. This tendency is still more independent of our consciousness of the degree to which we are aware of the fundamental forms of own thinking: concept, judgment, inference, because it is not often in fact that the object of our thought is our thinking itself. The introspection presupposes a second (deflected) *intention*, whereas our mental acts occur on average in the domain of *primary intention*.

Common thought draws the antecedent as a conclusion from the consequent, thus making the irregular inversion of *modus ponens*:

The rain makes the earth wet. Drenched. <hr style="width: 20%; margin-left: 0;"/> It was raining.	or	Wet is the earth. Drenched. <hr style="width: 20%; margin-left: 0;"/> It was raining.
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No formal logicians would accept the second as an inference if the first comes to their attention; they would see in it a perfect imperfectness. The concepts in premises are not distributed. “All,” “any,” and other quantitative words must be adjusted to them. Let us eliminate this failure and put its Aristotelian figure alongside.

All rain makes the earth wet. Here and now the earth is wet. <hr style="width: 20%; margin-left: 0;"/> It was raining.	P a M (Aristotelian S i M Fig. II)	<hr style="width: 20%; margin-left: 0;"/> S i P
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The situation is no better for the change. The Aristotelian figure or schema II must have at least one negative premise to be conclusive. And a negative premise gives only a negative



conclusion. In our example both premises and conclusions are asserting *a-i-i* is impossible in this figure. We give now a negative character to the second premise to obtain a valid syllogism.

All rain makes the earth wet. *P a M* (Aristotelian)  
Here and now the earth is not wet. *S o M* Fig. II)

—————  
Here and now there was no rain. *S o P*

This is already unobjectionable. It would be objectionable if some logician made the bond between word and concept too tight, as the phenomenologist Alexander Pfänder did. If we put two different expressions, instead of one middle term (wet earth—drenched), this could then give cause for objection.

But the place of occurrence of the incontestable is somewhere in the region where common practical thought frequently seeks the solution. It is in the relationships and interconnections between words and the concepts mediated through their meaning, concepts that reproduce, somehow or other, the internal connection of the phenomena relying on everyday experiences. In the contestable, doubtful short form of inference, practical consciousness unhesitatingly connects the two terms sharply set apart by formal logicians, the “wet earth” and the “rainfall,” in the concept of being drenched. It connects them unhesitatingly, because it is instinctively searching for the cause of the phenomenon. Examining real thinking and its forms, perceiving and understanding this disposition of everyday behavior and speech, Aristotle built up his theory of inference as mirroring the causal link in the middle term. Human activity is teleologically determined, but this teleology is subsumed into the objective determination of the conditions of life and subjective necessities. Scrutinizing the changes in weather conditions from the viewpoint of their own necessities, people bring together word families in their minds, trying to connect with the words and notions of wetness and rain those that, on the basis of their own experiences and experiences accumulated in the traditions of their progenitors, are important to the given situations, choosing and relating those that are generically or otherwise substantially essential for

the solution of their actual problems. And they know very well that there may be a tremendous difference between wetness and wetness. It is not only the knowledge of their trail-blazing ancestors, but of their most distant descendants. On this ontogenetical and philogenetical ground, the different or opposite concepts are reflected in each other, rain, or on another occasion a flood, in the concept of wetness, without knowing anything of the coordination and subordination of the concepts. To express it properly, they conceive them without any concept of the nature of their concepts.

***Difference between modes and figures of the Aristotelian and Hegelian syllogistic***

It is not enough to allude, as so frequently happens, to linguistic experience. Behind this level lies a deeper and hidden relation that I discussed already from the systematic point of view in my historico-systematic introduction to problems of Marxist philosophy (1984). None of the degrees of determination could be the negative identity of opposition unless it included its opposite, at least as a subordinated moment:  $I$  includes  $u$ . If it plays a peripheral part, let us bracket it:  $I(u)$ , the reverse  $U(i)$ , the median  $S(u,i \otimes i,u)$ , where we call  $u,i$  *commune*, and  $i,u$  *proprium*. How are these distributed in our example here?  $U$ , the concrete universal, is the earth, which includes wetness ( $u,i$ ) as the here-and-now element of its existence;  $S$  is the rain, sublating wetness in itself as the element of earthly existence;  $I$  is the here and now but not merely as a mathematical point, rather as a place of earthly existence. The wetness of a rain-drenched place is markedly different from the wetness of a flooded place. And it is for this reason that the wetness of the former is mentally connected with rain. If we have defined the ( $u,i$ ) of  $S$  as *commune*, and ( $i,u$ ) as *proprium*, then the preponderant side here will be *proprium*, the rain ( $i,u$ ). The mediating middle term is in this way *the rain-drenched spot of earth as U(niversal)*, including into itself, as a substantial moment of its general nature, the rain and the here and now. The whole question is embedded in the hydrologic balance of the Earth.

Every inference may be put down in a short form, having a double nature; it is a *modal judgment* and at the same time a *short inference*:  $I-S-U$ . But in our case, the mediating middle is  $U$ , non- $S$ , and therefore the short form:  $I-U-S$ . "Here and now in Rochester everything being drenched, it has rained."

In order to put our sentence into a form that contains an unambiguous inference, we must amplify it by double-setting the middle, and put a limiting point between the twice-occurring term. The second middle must be completed with the third term. In this way we already get the two premises of the inference. We separate the conclusion from the premises by a colon. The conclusion itself is nothing other than that the mediated extreme concept; i. e., the middle term in the conclusive judgment must be eliminated after its function is fulfilled. All that can be written in one row:

$$I-U . U-S : I-S$$

It is quite useful to know this. If you have a judgment  $S$  is  $P$ , and would like to ascertain whether it has a ground or it is without ground, or only based in extension, not in essentials, you try to put between  $S$  and  $P$  a middle term that will show whether or not it is adequate for the content. In the later development of logic as a propædeutic branch of philosophy, the row formula was broken up into a column, the period and the colon being used to fracture it.

But what we did here with the row formula, we may do in reverse with the column formula, beginning with the first, ending with the last, and elevating every section to the straight line of the row formula.

Here now is a diagram of the development of the figures of the syllogism without an example. Readers should be able to work it out without too much difficulty, taking the last example as a starting point. They must not make the mistake of confusing the "S" of the Aristotelian figure, meaning  $S$ (ubject), with the other "S" meaning  $S$ (pecificity) or  $P$ (redicate) with the other "P" meaning  $P$ (articular). This exercise is of course not necessary.

$$\begin{array}{ccc}
 & I-U-S & \\
 & I-U . U-S : I-S & \\
 I-U & S-U \text{ (Hegel III)} & P a M \text{ (Aristotle II)} \\
 U-S & I-U & S o M \\
 \hline
 I-S & I-S & S o P
 \end{array}$$

What is surprising in the third row, where the “modern” syllogism trestle work begins? First, in the Hegelian syllogism—and this is a much earlier device, Hegel simply conceding the tradition—the premises of the original (left column) are replaced. And second, the proposition major comes in the first place. *versa*. The reason for the change is so that the subject *S* (here = *I*) of the concluding proposition could be the *S* of the second premise, whereas the predicate *P* (here = *U*) would be that of the proposition major. This is one of the symptoms that the original concept-oriented syllogism has now become judgment-oriented, which, for topological reasons, unconsciously separates elements originally logically connected. The third point is Hegelian and profoundly affects the relation of dialectical and formal logic to each other. In Hegel’s logic, as in Aristotelian, three syllogistic figures are known. Whereas in Hegel, however, the third is the highest attainable, in Aristotle it is the second figure that gives only negative results, just as the Aristotelian third gives only particulars.

What is the ultimate ground of this tremendous difference, of the contradiction prevailing here? The mediating concept of the Hegelian Fig. III is the concrete universal having the moments of extremes in itself, which is, among other things, the strictly valid *modus ponens*, i.e., the recursive inference. We express clearly and distinctly in the language of modern science what in our example played the determinative part as universal: The Earth’s hydrologic balance (or “water-household”) is a cyclic process, closing itself and repeated again and again on different scales. This is the reason why the empirically and inductively conceived *modus ponens* becomes a *recursive inference* that by a firm step

goes back to the universal ground, and takes into account that the multiplicity of accidental cause-effect relations are no more than the forms of realization of a more fundamental ground-consequent relation. It is, of course, a commonplace mistake in the philosophical literature to identify in an abstract way the nexus of causality, which is an individual bond, in contrast to the general character of a ground-consequent relationship, with the nexus of ground-consequent ontological and logical categories. The best proof against this widespread mistake is that different causes may lead to the same result if these causes fall within the circle of validity of the same ground-consequent nexus. And inversely, the same immediate cause gives quite another result if it does not fall into the previously mentioned circle. The history of all the different social revolutions can demonstrate this. If it is necessary to know anything about the accidental mode of realization (which happens more often in the natural sciences), then there are always the probabilistic methods, as in quantum mechanics, where the parameters of the Hamilton-Jacobi differential equation are replaced by probability variables.

If the middle term of the inference describes only the quantitative relations of the surface, then, in its empirical immediacy, it cannot go beyond contingent assertions. But formal logic and its algebraic generalization are indispensable to this restricted cognition. The texture of empirical facts is from the point of view of ontology—or we may call it objective dialectics—in and by itself a contiguity of conflict between tendencies and countertendencies variegated by an almost infinite number of accidents. But we must not forget the well-known fact that fortuitous phenomena, or even their voluminous complex of whatever is associated with them, grow over into necessary and compact formations, and their deeper connections are graspable by means of statistics and the calculi of probability. With theoretical knowledge of these deeper, essential connections, we are able to form and influence those processes and occurrences that seemed at first to be beyond our effective power. One only has to think of the appalling frequency of lethal accidents resulting from the rapid growth of

traffic and of more effective counterbalancing measures. A deeper knowledge of real situations demands an examination of the broader and determining connections, not of isolated contingent cases.

One can never get a complete overview of empirical facts. This is not because of any insufficiency in our powers of inquiry, but due to the objective state of affairs. Individual cases of the same phenomena are usually so dispersed that it would be impossible to gather them in their totality, even in the given present. And with the present must be taken past and future processes. The individual cases of the past have once and for all disappeared. In this respect, those works that have a half-artistic, half-historico-sociological character are sometimes more fruitful in their description of events, as is the case with Thucydides in his description of the Athenian plague epidemic, or Defoe in the personal journal he left about the year of the plague in England, despite the fact that he had no personal experience with the events, not even having been born at the time. It does not matter, for the historic tableau he gives us is lively and convincing. By means of these general conceptions, we are in a better position to interpret and process the scanty information we have of those historical events than we would be without them. The fact that even the most stubborn empiricist is compelled to study only a partial group of the empirical facts actually available, using them as representative samples, testifies to the significance of the essential general viewpoints. It must be clear that a verified claim to empirical knowledge is no more and no less than one stage in the progress of cognition, which partly presupposes and partly gives a firmer basis, an effective means for producing theories and bringing forth essential conceptions.

We return now to the Hegelian Fig. III, which, according to the external arrangement, looks like the Aristotelian Fig. II, but which is in fact entirely different from that of Aristotle, having an important heuristic value in revealing essential relations, thanks to the concrete universality of the mediating middle term, whereas the formal Aristotelian treatment yields exclusively

negative results. Those who believed it to be inconclusive never understood the difference between the abstract and concrete universal. One of those, the philosopher and sculptor Betty Heimann, in having a good knowledge of Hegel's works, mixed Hegel with Bergson and worked out a false conception about which only the specificity, and not the individual or the universal, may be the mediator between the extremes of the syllogism (1927). Karl Rosenkranz, the reformist of Hegel's philosophy, whose reform experiments led to fatal, undialectical consequences that were convincingly criticized by Ferdinand Lassalle and Carl Ludwig Michelet—exponents of a well-equipped Hegelian orthodoxy—makes a still coarser mistake in returning to the Aristotelian order of figures (Rosenkranz 1858).

The Aristotelian figures are correctly named Stagirite (except figure IV, authentically called "Galenian"), after their discoverer. By their topological representation, the pedantic fracture of their structure, with each judgment written under the other, the characteristic placing of the middle term, the letters that denote the qualitative and quantitative character of the terms—all these are the inventions of later ages, aimed at setting apart theoretical study from common everyday thinking. In contrast to Kant's dictum on this, they have made some improvement.

***The concept "centrality" of the Aristotelian syllogism  
and the judgment "centrality" of his epigones***

The *Prior Analytics* (1990) of Aristotle is a formal logic, although both elements of this name are of late origin. But the work is formal logic (= analytic) because it takes as its foundation the extension and the subsumption relations of the concept. The concepts represent classes and subclasses, i.e., universals, particulars, and individuals, the last as the lowest species expressed in abstract individual terms, e.g., proper names, merely external analytic descriptions, even pronouns referring to individual objects. No development occurs, however, from one concept to the other; the different levels of concept are taken up as ready-made means from linguistic and empirical facts. This

complete absence of dialectics, especially in a great thinker with a strong dialectical disposition, can and must be explained by his anti-Platonism. His criticism of Plato's extreme idealism was partly connected to a skepticism against a high-flying dialectic. On the one hand, this was fruitful, making it possible for him to pursue the problem. On the other hand, it prevented him even in logic from elaborating his dialectical idea of the "causal theory" of the syllogism. Continuing the latter line, he could have recognized the meaning of the substantial middle term, and how the extremes are reflected in the middle and vice versa.

The absoluteness of genius and its earth-bound limitations appear here together—by no means always—although genius predominates. As did Boole, who, starting with a trivial form, recognized in it the possibility of a new branch of algebra at a time when algebra was the science of the solution of equations, so did Aristotle develop a scientific theory indispensable for one level of cognition out of the commonplaces of everyday thinking, blowing off the dust and bringing its elementary laws to the surface.

The figures of Aristotle are very near to the forms I denoted as the ways and modes of immediate practice-determined thought, the foundation of everyday thinking. This applies chiefly to the first figure, unfolded in the following way:

Whenever three terms are so related to one another that the last is contained in the middle as a whole, and the middle is either contained in, or excluded from, the first as in or from a whole ["Here all three terms are frankly treated in extension," says the well-founded commentary of W. D. Ross (1923, 34)], the extremes must be related by a perfect syllogism. I call that term middle which is itself contained in another and contains another in itself: in position also this comes in the middle. By extremes I mean both that term which is itself contained in another and that in which another is contained. If *A* is predicated of all *B*, and *B* of all *C*, *A* must be predicated of all *C* [Fig. I, mode



of value-distribution *a-a-a*—my comment, J. S]: we have already explained what we mean by “predicated of all.” (Aristotle 1990, 25b32–35)

In the same work, he understood “predicated of all” in this way:

That one term should be included in another as in a whole is the same as for the other to be predicated of all of the first. And we say that one term is predicated of all of another, whenever no instance of the subject can be found of which the other term cannot be asserted: “to be predicated of none” must be understood in the same way. (24b28–30)

Further on, Aristotle writes:

Similarly also, if *A* is predicated of no *B*, and *B* of all *C*, it is necessary that no *C* will be *A*. [Fig. I mode; *e-a-e*—my comment, J. S.].

But if the first term belongs to all the middle, but the middle to none of the last term, there will be no syllogism in respect of the extremes.(26a1–6)

Let us return to the first and shortest scheme of syllogism, to the tripodal one, but now in the spirit of Aristotle, whose system of syllogism was built on the relations of termini. The characteristic feature of his theory is the concept “centrality.” To get a symbol for the following representation of the original Aristotelian syllogism, which was a proper relation between the termini, we shall write one adequately chosen letter in the attribute of termini, using it to indicate the order of termini in the syllogism: mInor, miDdle, maJor term = *I-D-J*. We put down this scheme in two forms, ( $\alpha$ ) and ( $\beta$ ), with the illustrating concepts below showing how it is to be interpreted.

	<i>J</i>	–	<i>D</i>	–	<i>I</i>	( $\alpha$ )
	mortal	–	human	–	Greek	
or in another order	<i>I</i>	–	<i>D</i>	–	<i>J</i>	( $\beta$ )
	Greek	–	human	–	mortal	

Connecting the two extreme concepts in such a way that we refer the major term, *mortal*, to the minor term, *all Greeks*, we get the following Aristotelian assertion (inconvenient to our eye and ear accustomed to judgments like *S is P*): “mortality refers to all Greeks” or “mortal is the predicate of all Greeks,” or according to our present usage, “all Greeks are mortals.” This is a quantitatively universal, qualitatively positive judgment. The judgment in its isolation is not the conclusion of an inference. But on the other hand, there is no judgment without a claim to truth. The necessity for a sufficient reason arises willy nilly. Therefore we must go to the amphibious logical form, to the modal judgment, which is already an inference, a short one. In this short form there is and must be a place for a middle term, which in the present case will be *human*: “all Greeks, being human, are mortals” ( $\beta$ ). Or in the Stagirite style, “mortality, as to every human, refers to every Greek” ( $\alpha$ ). The middle term is on the one hand more extensive than the concept of a Greek human, and on the other hand is subordinated to the totality of mortals. And this already relates back to an ontological fact, to the causal link between the human nature of Greeks and the mortality of humans.

In the final analysis, this logical necessity depends on an ontological necessity, although there are many links of abstractions between real necessity, such as objective and logical necessities as a subjective category. The short form, however, does not demonstrate the nature of the connection absolutely clearly. This can be rectified by double-setting the mediating middle, the concept of *human*, not on a whim, not even out of respect for clarity, but owing to the objective circumstance that the concept *self* is divided. One thread goes from humans downward to ethnic groups, to the Greeks; the other upward to living beings in general from human beings whose fate and mortality, in contrast to the immortality of Greek deities, was so captivantly described by Hölderlin in the poem, “*Hyperions Schicksalslied*,” from his lyrical novel pervaded with nostalgia for a past fuller life, where at least the deities knew and experienced what a rich totality of life means.

Doch uns ist gegeben  
Auf keiner Stätte zu ruhn,  
Es schwinden, es fallen  
Die leidenden Menschen  
Blindlings von einer  
Stunde zu andern,  
Wie Wasser von Klippe  
Zu Klippe geworfen  
Jahrlang ins Ungewisse hinab.

And Hölderlin speaks wonderfully of the immortals, although we no longer believe in them :

Schicksalslos, wie der schlafende  
Säugling, atmen die Himmlischen,  
Keusch bewahrt  
In bescheidener Knospe,  
Blühet ewig  
Ihnen der Geist,  
Und die selige Augen  
Blicken in stiller  
Ewiger Klarheit.

The strophe gives only the possibility of a negative conclusion, even in embryonic form. A human being, not being a god, is not immortal. And Aristotle's form that doubles the middle term tells us the thing, "No human is due to the predicate god, all gods are immortal, no human is due to the predicate immortal." Every short inference may be completed by the double positing of the middle term, regardless of whether it is written in a row or in a column. (The Greeks never used a column, not even for verse, despite Łukasiewicz's blunder in his book, *Aristotle's Syllogistic from the Standpoint of Modern Formal Logic* [1951], who by this mischance proves that he never glanced at the original Aristotelian text.) Let us demonstrate step by step this development, without historical concretization.

$J$          $-$          $D$          $-$          $I$   
mortality     $-$     as to every human     $-$     refers to every Greek

$J - D . D - I : J - I$

mortality refers to every human . human refers to every Greek :  
therefore mortality refers to every Greek

We shall now break up the row formula into a column, getting Fig. I. We write it first according to its original (non-Greek) quasi-topological form, then beside it, in “modern” form. Left and right of the assertions we put the “Aristotelian” and the “modern” usual schemes.

$J u D$	mortality refers to every human	every human is mortal	$M u P$
$D u I$	human refers to every Greek	every Greek is a human	$S u M$

---

$J u I$  mortality refers to every Greek    every Greek is mortal     $S u P$

The primary scheme, Fig. I, has so great an advantage over the “modern” that the quasi-topological double-setting of the middle may be seen in immediate succession, even in the form of a colon. It is not only a sign of how near this form is to practical thinking, but clearly shows—not so clearly, however, as the row formulation—Aristotle’s orientation on the concepts. For him the last “atomistic” unities were the concepts, putting them into a hierarchical order, for it is an external relationship, and has no deeper meaning in that of “inclusion.” The later, more modern, formula at least paves the way toward judgment “centrality” and a post-Aristotelian idea. The subject unfolds itself in the predicate, but their connection has no clear-cut grade of determination. Its accidental or necessary character must be expressed by a special kind of judgment. This already had its beginning in Aristotle, who described it with a separate modality index. They lack almost absolutely the expression of internal necessity that must follow from the development of the subject–predicate relationship.

***Modality index or dialectical modality developing itself  
in the changing relations of the terms of inference***

But is there any real value in the external mechanical device for the theoretical understanding of the logical structure of the thinking process? Hardly. What does it mean if we write out in every judgment of a syllogism: Every human is necessarily

mortal—Every Greek is necessarily human—Every Greek is necessarily mortal. It is hardly more than a merely subjective assertion about the necessity of human mortality. Whether we are believers or not, we have not the slightest idea about the real foundation of human mortality. Everybody knows as an empirical fact that all human beings must one day die, and yet perhaps there is no illusion more widely spread than that of a life after death, be it the Christian heaven or the otherworldly prairies of the North American Indians.

We must look at logical problems from the real categories of ontology, from the ontological dialectics of chance and necessity, where the necessary tendencies, beginning with accidental individual changes, grow into phenomena of longer or shorter duration in society as well as in nature, determining the seemingly chaotic play of individual cases.

Aristotle from time to time runs up against the problem that the category of extensive universal covers only the major part of the individua of a genus or species, and the minor part is hardly or not at all classifiable. But he is not able to draw its true consequences, even though, in the lines of his theory of the causal dependency of the middle term, he makes some steps forward, being a thinker of the type who, when touching on a problem, always says something importantly original. Fortunately, his nonproverbial, contradictory antipathy toward the Heraclitian and Platonic dialectics prevents him from taking into account the very dialectical nature of the middle term, which, not in its *medius*, but in its *medians* nature, in its mediating character, is nothing else than a *contradiction* proceeding *on two feet*; in principle, he does not take into account the inference process of *I-S-U* or *U-S-I*, where the dividing-into-opposition and unifying-after-division opposition turns into *conciliation* [*Versöhnung*]*—a process much more characteristic of thinking than for real processes, because in the latter, as a rule, or even as a law, the reverse takes place: the conciliation turns usually into antagonistic oppositions.*

Let us look again at the preceding explanation as to what kind of middle would connect, not as a subjective assertion, but as an objective necessity, the extreme concepts of the syllogism.

Surely, the answer is the kind of middle that would fulfill the requirement not only of necessary but also of sufficient reason, i.e., a dialectically concrete middle. A human being is not mortal in general, but as a living being, although there still remains the problem that the special trait of such a being is its social character, meaning that in proportion to the significance of his or her objectifications, the results survive. In contrast to Nicolai Hartmann's theory about everlasting life, built upon the theory of Weissmann's germ-plasma, and in contrast to a similar theory of Bergson that dematerializes the plasma into a "spiritual energy," human beings do not survive but their deeds do. As natural individuals they disappear forever. The judgment "every Greek is mortal"—in spite of the specific social signature—is mediated by the concept of a living being. All Greeks as living beings are mortal. There we have the cause of their mortality.

This is, on the level of logic, a solution of the problem. But this solution compels us to a theoretical amplification. The assertion that every living being is mortal still contains the empirical inexhaustibility of living beings as sketched above. And in this way, it puts before us a Kantian never-ending approximation process, never giving a real certitude. *Our judgment, intended to be universal, reveals itself as particular.* There remains a considerable distance between living beings and mortality. We can take it as the same as the distance between living beings and Greeks. Living being as a middle term may associate mortality with the goats pasturing on the vegetation of the Greek mountains, just as well as the goats pasturing on grass. If we want to take logical empiricism seriously, then formal logic, together with its empirical orientation, leads us to a polysyllogism that can proceed in two directions, upward and downward to infinity, from which I give a section:

- Every living being is mortal.
- Every human is a living being.
- \*Every human is mortal.
- Every Greek is human.
- \*\*Every Greek is mortal.
- Every Spartan is Greek.
- \*\*\*Every Spartan is subject to death.

Each judgment marked with an asterisk is the conclusion of the previous premises and the first premise of the following conclusion. All this does not eliminate the difficulties; it just makes thinking more clumsy and tedious. The only kind of theory that can help is one in which life and death are dialectically united in a process of development and not empirically separated data in correlation, although we can place on one side all our dead contemporaries, and on the other, all those still living in their utmost alienation from each other. Both sides remain in movement, in never ceasing rearrangement; bodies in atomic dissolution in nature; living human beings in social, generational, and vocational replacement to and fro, up and down. Brisk life is the unity of opposition; one pole sweeps across the other, although at a different time some more or less stable proportions always exist. Mortality in such a theory expresses the specific moment of life; there are plenty of differences in how the individuals of ontological strata pass away. In the life process, death and life belong together in every moment. The essential judgment tells out this unity of opposites: "A living being is mortal."

Such judgments Aristotle holds mostly as *indefinite*, although they are only quantitatively indefinite. *Compared to quantitative definiteness, it is essential definiteness.* Spinoza did not speak about *omnis* when he defined *determinatio* as *negatio*, because he spoke about the essence of determination *pur et simple*, leaving open the question whether in every concrete case this negative movement is validated in a posited form. He knew that the quantitative definiteness—this may be proved from his other writings—cannot be the decisive moment because a new phenomenon may be essential even though not, or not yet, widespread. And if already widely spread but externalized so as almost to lose its essence, then it may be on the verge of destruction. Something like this has happened in the destruction of "existing socialism," and in a concrete analysis of historical fact, this point of view would be useful in clarifying the deeper-lying causes, the fundamental processes of this destruction. And not only that, but the manner and method of a new upsurge.

We spoke about mortality as one essential determiner of living beings. Its degree of determination must be *U*(niversal). The *S*(pecificity) must be adjusted to the living being from the three fundamental movement forms of nature; the second, under given conditions, grows into the societal, individual-movement form *I*(ndividual) of nature as its most complex phenomenon that, owing to its extremely complex character, probably very seldom comes into existence in the whole of nature. When speaking about one part of humanity, about Greeks, the middle term must already have this level of concreteness, which is not attainable without theory. The living being alone, as the middle term, does not bring us to Greek human beings. From the concept of living being, by means of theory, we need to get another essential concept or other concepts. Such a one may be the concept of living being characterized as social and historical, let us say the *zoon politikon*, as realized in antiquity paradigmatically, in the destiny of the Greek people. In the context of such considerations, the syllogism looks like the this: “Social living beings and their populations are mortal. The Greeks as individuals and as a people are social living beings. The Greeks are mortals.” The middle term here functions more concretely. But it is still a long way from being completely concrete. This tendency toward completeness makes us aware of the gap, a not-yet-formulated problem, and in the process of cognition this gives a strong stimulus. This stimulus is lacking in the Aristotelian counterpart of the previous Hegelian form.

The much-criticized Hegelian structure is better than the Aristotelian, because the grade of determination of the concepts *minor*, *middle*, and *major* reveal the part they play in the syllogism when we replace the initial middle with the extremes. Each concept will be the middle and mediating terms in all three figures. Concepts and judgment are just moments of the syllogism, which is an articulated whole. The diagrams below give a comprehensive and visually comparable picture of the three Hegelian and Aristotelian figures with the short form left and right. (As an example we may use: “Living beings are mortals / The Greeks are living beings,” etc.)



$I-S-U$	$S$ is $U$	Hegel I	$M$ is $P$	Aristotle I	$S-M-P$
	$I$ is $S$		$S$ is $M$		
	$I$ is $U$		$S$ is $P$		

The Hegel Fig. I gives as its conclusion  $I$  is  $U$ . This will be the starting proposition of his Fig. II, which is in Aristotle Fig. III.

$S-I-U$	$I$ is $U$	Hegel II	$M$ is $P$	Aristotle III	$S-M-P$
	$I$ is $S$		$M$ is $P$		
	$S$ is $U$		$S$ is $P$		

Comparing Figs. I and II, we can see that the conclusion of Hegel Fig. II is the same as the starting proposition of Fig. I. We begin to see that the syllogistic figures are *internally* connected. The conclusion of Fig. II deduces the starting premise of Fig. I. It proves that the premise has a solid ground. Now there is only one premise, the second of the first, which is still without ground. Continuing the method and using the last conclusion as the starting proposition of the Hegel III syllogism, we can attain this solid ground. We may understand now why Hegel put this figure in the last place.

$I-U-S$	$S$ is $U$	Hegel III	$P$ is $M$	Aristotle II	$S-M-P$
	$I$ is $U$		$P$ is $M$		
	$I$ is $S$		$S$ is $P$		

With the last step, the Hegelian syllogism is closed. Both premises of Fig. I are now grounded. On the other hand, Fig. I, mediated by II, gives ground to Fig. III. If we only take their formal interwoven configuration into account, then it must be clear that this is not an external classification of inferences as that of Aristotle but an *organic system* that may reproduce itself in more amplified forms, forming itself into a theory after a number of ramifications from expected and unexpected turns, from relapses and from the great upsurges as well. It is a subtle "mirror," a

spiritual reproduction of the multidimensional processes occurring in the reality.

***The inference “centrality” of Hegel’s logic and the “master syllogism”***

Hegel created a multidimensional logic, some points of which were later discovered independently. Such was DeMorgan’s break over the rigid opposition of contradictory and contrary opposition. In the German development of logic, some values were adapted from him directly, mostly in an isolated form destined to serve conceptions entirely foreign, if not inimical, to dialectics. It is true that most of the successors were engaged in experiments to reduce the whole stock of syllogisms to the different variations of *modus ponens*. This was done partly after the example of the pre-Hegelian German classics, after Kant 1762 and Fichte 1812 (who in sharp contrast to Hegel, imagined the richness of the figures of syllogistic thought to be hair-splitting pedantry). Hegel discovered in them one important stage of the historical development of the cognitive process from empirical facts to the highest level of the essential dialectical connections. He tried to disseminate it. And this was a productive dissemination. It was not his fault that the seeds fell on fruitless, stony soil, after the frustrated German *Vormärz*.

The highest plateau of logical development is the area in which all forms of inferences integrate their degree of determination mediated by their general law. But this is not, and may not be, on the lower levels. In our last example (“The Greeks as living beings are mortals”) the *U*, in spite of its generality, does not play the part of an essentially concrete mediator, not being invested with the necessary content. The *S*, on the other hand, has the necessary content, but it does not attain the optimum whereby it could capture both extremities; it captures only one. But what does this optimum mean? And how can it be proved? It means that each concept, when playing mediator, must capture the extreme degrees of determination, without any shortfall, i.e., without falling back to the relations determined by mere extensionality. Out of this follows the proof: in the process of

inference, subject  $S$  and predicate  $P$  of every judgment must be interchangeable,  $S$  is  $P = P$  is  $S$ . This may be written in the way given when speaking about the internal structure of every degree of determination, which has its opposite in itself  $U(I)$  etc. with or without parentheses:  $S(p)$  is  $P(s) = P(s)$  is  $S(p)$ . This is what I call the proof of dialectical commutativity.

Taking “The Greeks as living beings are mortals” in its unfolded form. The first premise will be (see Hegel I on page 362): “The living beings are mortals.” Is this sentence commutative? Yes it is. The mortals are living beings. And the second premise: “The Greeks are living beings.” Can we swap  $S$  and  $P$  and still retain validity? Living beings are Greeks. This is now false. There are a good many other kinds of living beings, people as well as animals. The second premise slides back from the height of essential connections of specificity to mere particularity: Some of the living beings are Greeks. (The case with the conclusion is the same for the same reasons.) If we want to get nearer to a more concrete determiner of specificity, then we must take into account the other next determiner “ $a \otimes i$ ,” the other side of  $S$ , say, “ $i \otimes u$ ” = *zoon politikon*. Then the mediator would be: living being as *zoon politikon*. But this now requires a theory for different reasons. This type of problem is what Rosenkranz could not understand, which does not mean that for keener-minded Hegelians like H. Hinrichs, Carl L. Michelet, not to mention Ferdinand Lassalle, this problem would have been alien.

But without making any further inquiry, let us exemplify this highest level of logical thinking, using the system of syllogism that expresses, at least partly, Marx and Engels’s well-known theory about human beings. This system is not only a summing up of theory, but in many ways an indispensable means of cognition in the different stage of working out a theory. Such was, for example, the role of that concept and judgment in theoretical clarification and verification that was derived from Benjamin Franklin’s famous definition of man as a “tool-making” animal. We shall see the intertwinement of the dialectical figures I–II–III, where, seen from the totality of the system, Figs. I and

II are quasi-premises for the conclusion of Fig. III. The mutual replacement of *S* and *P* in this case is perfectly valid. We do not write it out; readers may do this themselves. On the basis of what we have already stated, they will be able to do so without much difficulty, making sure that they remain concrete identities and a valid syllogism in their inverted state. But they must not believe that this form is a negligible quantity. It is a useful and usable form if one is interested in the question of how universals penetrate individuals. You must always invert!—to recall Jacobi’s dialectical advice. The conclusions marked with asterisks are always the first line of the new syllogism. The direct forms in their intertwinements are:

Fig. I.  $I-S-U$     $S-U$    The tool-making animal is a social being.  
                    $I-S$    Man is a tool-making animal.

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Fig. II.  $S-I-U$     $*I-U$    \*Man is a social being.  
                    $I-S$    Man is a tool-making animal.

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Fig. III.  $I-U-S$     $**S-U$    \*\*The tool-making animal is a social being.  
                    $I-U$    *Man is a social being.*

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$***I-S$    \*\*\*Man is a tool-making animal.

Concepts and judgments made by the mediation of inference mutually support each other. And conversely, concepts that arise at the same level strengthen the truth of the syllogism. The first premise of Fig. I was already strengthened by the conclusion of the second syllogism. And the second premise obtains solidity and strength as the conclusion of syllogism III. It strengthens the middle concept of the beginning, a cyclical process that realizes itself, and which is closed and at the same time open. It allows us to go further in different directions, making new cycles in order to discover, e.g., what is the exact meaning of the concept of “tool” in contradistinction to the tools of the animal world. In such a way it must be regarded as an open system, having only relatively closed parts, *relative totalities*, which may be absolute at a certain time and for a certain duration, that is, after attaining

a relatively higher level. Their results compel going further. In spite of all these necessary connections and criteria, a well-chosen middle term to begin with and derived at the end of the system is in itself no guarantee for truth. In the same way we can conclude with no logical flaw, using a false but concretizable middle term, that human beings are a religious, or a playful being, a *homo ludens*. And these are the sources of human culture. The first is a fundamental element of the clerical worldview, the second that of the postmodern (although Thorstein Veblen already criticized it in his *Theory of the Leisure Class*). A real dialectical middle term must materialistically verify its deep roots in the fundamental contradictory relations of the given reality. It must be a *genetical-generative concept*, being capable of ramifications in different directions, explaining their mode of existence. This is why I have several times emphasized the necessity of theory-building. Hegel was very close to the realization of this. But to be near means a distance too. One important sign of his nearness is how seriously he took the starting point of the syllogism. In seemingly complete harmony with Aristotle, he put down the sentence: “*E–B–A* ist das allgemeine Schema des bestimmten Schlusses. Die Einzelheit schliesst sich durch die Besonderheit mit der Allgemeinheit zusammen [*I–S–U* is the general scheme of the determined inference. The individual closes itself with the universal through the specific.] (1936, 5:122). This harmony with Aristotle is less complete than it seems, because Hegel, despite his not having a common name for them, uses degrees of determination.

The third syllogistic figure, at the highest point of development, is what I should like to call the *sylogism of sylogisms*. It is easy to demonstrate why. It is enough to put together the conclusions of the different syllogisms (marked by asterisks) in a syllogistic order and we get the third figure, only the two premises changing their place with each other.

Fig. III. *I–U–S*    \**I–U*    \*Man is a social being.  
                   \*\**S–U*    \*\*The tool-making animal is a social being.

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\*\*\**I–S*    \*\*\*Man is a tool-making animal.

But there is another denotational possibility too, even better, because it shows that this form is something new. It may also be called: the *master syllogism*, being an abbreviated form, a recapitulation, but also an anticipation of the main stages of logical development. This recapitulatory and anticipatory character has not only analogy, but structural community in music, poetry, practically all branches of art, even those which sublimate their diachronism into synchronism. One example will do for all. One of the most artistically chiseled forms of poetry is the sonnet, a poem of fourteen lines. A series of sonnets may be refined still further. By writing fourteen sonnets you can build up a cyclic form. Each subsequent sonnet begins with the last line of the preceding one. The last sonnet, the fourteenth, concludes with a line identical to the first. Alpha and omega come together. But not unconditionally. From the identical last and first lines of the preceding fourteen sonnets you can write a fifteenth, the *master sonnet*, which will be old but at the same time new. This is the real  $\alpha$  and  $\omega$  in a twofold sense. You may put it in front of the whole cycle, as a *prelude* followed by the analytic unfolding of its content. Or it can go at the end as well, as a *postlude*, a synthetic enfolding, a wrapping up of what has gone before.

The same may be done with the master syllogism, using it in an anticipatory way as a heuristic means of inquiry or as a final transillumination and systematization of an examined material. *It can be used as either one or the other, or both together, because it is the logic of scientific discovery.*

This leads us to questions of scientific methodology, which is not an appendix of logic, but an important organic part of epistemology. It is enough to mention that Marx, with supreme knowledge and with his materialistic corrections, used it as such in the introduction to *Outlines of a Critique of Political Economy (Grundrisse)* (1986). In the general theoretical notions of bourgeois economists, Marx discovers the *logical guiding principle*, which is of necessity *an inference*.

Today when every conception, especially a general conception, means *an intuition, a vision, an intuitive vision and a visional intuition, without logic*, nobody dares to say things like

this, not even the more rational thinkers, as a result of the widespread and fashionable irrationalism. To be sure, we may accept Einstein's confession that his theory of relativity has as its personal kernel his youthful experience of being captured by the problem of whether there is a possibility of overtaking a ray of light. This may be regarded as a kind of vision. But first, it was a vision of a young man interested in physics, which is unimaginable without intellectual effort. Further, if the problem remained on that level, we would not have today the special and general theories of relativity.

The analysis of Marx begins with the following statement:

Production, distribution, exchange and consumption thus form a proper syllogism; production represents the general, distribution and exchange the particular, and consumption the individual case which sums up the whole. This is indeed a connection, but a superficial one. (1986, 27)

Elsewhere he points to the lack of a concrete genetical-generative concept, by means of which bourgeois economists could have arrived from the general relation of production to private production: "But it is ridiculous to make a leap from this to a definite form of property, e.g. private property" (25).

Why is all this so superficial, even ridiculous? The basis of the superficiality is nothing other than the predominance of external, extensional logical relations, which Marx called "*a speculative connection*," in contrast to essential logical connections of the degrees of determinateness. (Marx did not use the term *degree of determination*.) And he goes forward by means of these essential determiners:

If these trivialities are reduced to their real content, they say more than their preachers realize, namely that each form of production produces its own legal relations, forms of government, etc. *The crudity and the lack of comprehension [lie] precisely in that organically coherent factors are brought into haphazard relation with one another, i.e., into a merely speculative connection.* (italics added). The

bourgeois economists only have in view that production proceeds more smoothly with modern police than, e.g., under club-law. They forget, however, that club-law too, is law, and that the law of the stronger survives, in a different form, even in their “constitutional State.” (25–26)

Marx not only criticizes the conceptual deficiency and the externalization of inference of the bourgeois theories, but shows positively, giving an exceptional paradigm, how to use the logical forms of dialectics as a means of scientific discovery (in the whole of the Introduction); for example, how the dialectical concepts (private property—not private property—common property) go into each other; how much the judgment in its immediate dialectical contradictoriness (“production is thus directly consumption” [28]) and—as if he had heard the advice of the mathematician Jacobi: always invert to get new results!—its inverted form (“consumption is directly production”), which is the oppositional form of the previous assertion. This may and must be extremely fruitful by virtue of its compulsion to think everything through to the very end, that is, to grasp everything in its articulated totality: “Each is immediately its opposite. At the same time, however, a mediating movement takes place between the two. Production mediates consumption for which it provides the material. . . . But consumption also mediates production, by providing for the products the subject for whom they are the products” (28). The mediating process between the opposites is physical human activity led by an idea adequate to human and social needs. In intellectual production, the mediations between the opposite poles are realized by the changing middle terms as *a mediating middle in the processes of inference*. Marx, in his paradigmatic analysis, sometimes writes out the premises, sometimes not, but the presuppositions in these cases are also always reproducible, comprehending the whole in its internal articulation as a multiplex unity of a continuable and further concretizable open system, being able to grasp the ever-changing world, which does not always happen and is not always free of error.



***Identity-process and contradiction-progress***

In my exposition, I put the logical consequences and ramifications of the principle of identity of identity and not-identity into the foreground. (I have taken the word *principium* in its objective Greek meaning, as *arche*, arch = primordial, beginning, moving power, and not in the subjective Latin sense: *principim* = primum capere, catching the first.) And I was compelled here to a sensible one-sidedness that does not delineate in a deeper concreteness those fundamental ontological categories that, mediating each other with human activity and praxis as a unique social activity, imprint themselves on the human mind as a motivating system of behavior, on a level more or less adequate to the comprehensiveness and depth of the prevailing societal praxis and activity. (The fundamentals of this conception I have already detailed in *The Revolution of Scientific Thinking* [1984, 199–251]; see part 4: 1. “Roots of contradictions in the *Weltanschauung* in the process of material production”; 2. “The formal process of thinking and social production,” (α) The problems of degrees of determination and mediation processes, (β) Mediation processes and forms of inferences.)

These forms and ways of thinking come into operation in every sphere of social activity—mostly in the specific and historically changing forms of the given spheres—in theoretical spheres as well as in artistic, in ethical as well as political, no matter whether in secular or clerical colors. This is a fact that many people often fail to take into account, and with good reason, for if they did, their whole irrationalist creed would collapse like a house of cards. The new Hegelian principle, in a certain sense not without materialist corrections, proves its validity in the development of the mental sphere, which seems to be relatively independent. An absolute independence, however, is no more than an objective-idealist illusion, because mental development, despite false appearances, when some sort of breakthrough becomes necessary, makes this transcending step not merely as a result of the accumulation of its own unsolved internal contradictions, privations, and insufficiencies, but as a consequence of the

emergence of new historical constellations. Whereas Hegel in his principle of identity and not-identity took identity as absolute, Marx took contradiction as absolute and identity as relative.

We must not forget, however, that Hegel did not conceive his synthetic identity as an abstract one—he vigorously refuted Schelling’s *Identitätsphilosophie* as an irrationalist mystification—but as the law of reproduction on the same level. As a consequence of this, the dialectics of relative and absolute must have their own specificity, even in connection with the Hegelian principle. Expounding this would lead us into a new area of problems *following process-identity to progress-contradiction*, which must bear the hallmark of Karl Marx. As Marx went from the labor theory of value, making an indelible historic step forward, to the labor theory of surplus value, and from this higher point of view correcting important parts of the foundation, so must we, having made some significant points, continue the necessary corrections with the exposition of the theory of progressive contradiction.

Translated by the author with minor modification from the Hungarian original “Azonosság és nem-azonosság azonossága,” *Magyar Filozófiai Szemle* (Budapest), nos. 4-6, (1988):435–507.

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#### NOTES

1. A Russian translation of two chapters of Szigeti 1984 containing most of the material of this book referred to here can be found in Szigeti 1983–84.
2. It is important to know the critical part of Spinoza’s letter:

With regard to the statement that figure is a negation and not anything positive, it is obvious that matter in its totality, considered without limitation, can have no figure, and that figure applies only to finite and determinate bodies. For he who says that he apprehends a figure, thereby means to indicate simply this, that he apprehends a determinate thing and the manner of its determination. This determination therefore does not pertain to the thing in regard to its being; on the contrary, it is its non-being. So since figure is nothing but determination and

determination is negation, figure can be nothing other than negation, as has been said.

3. Brouwer was leader of the tendency in research on the fundamentals of mathematics that is mistakenly called “intuitionism.”

4. I agree with the critical remark of Russell that “intuition” as “the accepted translation, is not altogether a satisfactory one” (1947, 708), dismissing at the same time Russell’s attempt of solution.

5. All translations from non-English sources are made by the author.

6. There were some difficulties in the acceptance of negative numbers as such. Through the centuries, they have been regarded as lacking legitimacy. Even the marks “+” and “-” were only born in the medieval warehouses, chalked on sacks to signify whether they exceeded or fell short of the weight assigned. After use in commercial arithmetic, they became integrating symbols through the “pure” mathematics of Vieta-Stevin-Descartes in the sixteenth and seventeenth centuries.

7. Boyer, who is not interested in the philosophical background of the history of mathematics, writes in his otherwise meritorious work:

The most striking property of this new higher transcendental function was . . . that in the theory of complex variables they have a double periodicity, that is, there are two complex numbers  $m$  and  $n$  such that  $v = f(u) = f(u + m) = f(u + n)$ . Whereas the trigonometric functions have a real period only (a period of  $2\pi$ ) and the function  $e^x$  has an imaginary period only ( $2\pi i$ ), the elliptic functions have two distinct periods. So impressed was Jacobi with the simplicity achieved through a simple inversion of the functional relationship in elliptic integrals that he regarded the advice, “*You must always invert,*” as the secret of success in mathematics. Jacobi deserves credit also for several critical theorems related to elliptical functions. (1968, 556–57)

8. Tsereteli was a professor of philosophy at the University of Tbilisi, having been a soldier on the German and Japanese fronts.

9. It is perhaps worth mentioning that the term *triad* is not Hegel’s own invention. He took it from the neoplatonist Proclus—a significant thinker highly above average, with erudition and acumen—who used it frequently; for example, in *Théologie Platonicienne*, he wrote, “These triads come down from the most primordial and principal causes, as we disclosed in the analysis of the preceding triads” (1968, 27–29). But whereas Proclus used the triad in the spirit of the number mysticism of the later Plato, for Hegel it is a mere technical denotation.

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## BOOKS AND IDEAS

by *Herbert Aptheker*

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### **The passing of a hero**

Pham Van Dong, the sterling leader of the Vietnamese liberation struggle, died on April 29. He was 94 years of age; he began his revolutionary career before his twentieth birthday as a leader of student protests against French rule. For many years he was a political prisoner. Decisive in the struggle for liberation, he was a comrade of Ho Chi Minh.

Pham Van Dong was perhaps six feet tall and darker than the ordinary Vietnamese. When, as a visitor to his country in 1965, I remarked on this, he said yes, that his ancestry was from Borneo. On a stroll about some government buildings, he stopped before one (now a museum), and said he had stayed some years in prison there. It then struck him that I faced possible imprisonment on my return home. Perhaps you should stay here, he suggested. But then, recalling the purpose of my visit to Vietnam—to urge the ending of Washington's brutal intervention—he said, of course that is absurd. But he wished me well.

I remember saying to him that some madmen back home, including U.S. senators, were urging the use of atomic weapons. He knew that, he said, but he added that he did not think this would occur. True, Washington knew that Vietnam had no such weapons. But, he went on, it also knew that Vietnam's friends—meaning China and the USSR—did have them. Thus he hoped to be spared such an ordeal.

Pham Van Dong's serenity struck me. Since the 1920s he had devoted his life to national liberation and socialism. Now, in the

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midst of his nation's torment, he was certain of ultimate triumph—and actually concerned for *my* well-being!

### **The passing of a mass murderer**

On 30 April 2000, Martin Schilling died at the age of 88, entering hell from a clinic in Burlington, Massachusetts. He had been a leading member of the team working for Hitler that devised the V-2 weapon—the *V* standing for *Vergeltungswaffe* or Vengeance Weapon.

The *New York Times*, in a full obituary May 6, noted that the V-2 emitted no warning of its approach. It could not be aimed with precision, the *Times* said, and was “considered a terror weapon.” About one thousand of these devilish devices were fired towards the vicinity of London, and about four thousand against Allied soldiers—one of whom is writing these sentences.

Schilling's son is quoted in the *Times* as remarking that the weapon in flight offered “a beautiful picture, showing the tiny rocket against the immensity of space.”

At the close of World War II, 126 Nazi scientists who had developed these weapons were settled in Fort Bliss, Texas, by the U.S. government. There, writes the *Times*, “they continued to develop rocketry” and intensified the weapons war against the Soviet Union. In 1958 Schilling was awarded the “Exceptional Civilian Service Award of the U S. Army.” Late that year, according to the *Times*, Schilling became part of the Raytheon Company, now “one of the world's largest electronics and missile system contractors.”

Earlier, in 1941, Schilling had risen to technical director of Hitler's organization developing rocket weapons. The article in the *Times* does not say whether or not Schilling's Raytheon Company employs Jews.

### **A thwarted right-wing coup**

The attempt to unseat President Clinton represented a reactionary conspiracy to seize control of the government of the United States. It was thwarted thanks to the President's resistance, the remarkable steadfastness of a deceived and yet loving

wife, and the intelligence of the majority of the U.S. population, who understood the deeply reactionary purposes of the entire sordid plot.

The reality of this criminal effort to seize control of the government of the United States is fully documented in two important books: *A Vast Conspiracy: The Real Story of the Sex Scandal That Nearly Brought Down a President* by Jeffrey Tobin (Random House, \$25.95) and *The Hunting of the President: The Ten-Year Campaign to Destroy Bill and Hillary Clinton* by Joe Conason and Gene Lyons (St. Martin's, \$25.95). The story is carefully analyzed and brilliantly evaluated by Anthony Lewis in a long and vital essay in the 13 April 2000 *New York Review*.

The right-wing conspiracy involved the supermarket sex sheet, the *Star*, and, in its origins, the support of Jerry Falwell's so-called Liberty Alliance. Using a lying scandal video, it accused the President of crimes ranging from being a drug runner to an abettor of murder. This video was shown to uncounted thousands through the influence of Falwell and something called Citizens for Honest Government, which said it sold 150,000 copies of the tape at \$40 each.

Falwell's efforts were supported by millions of dollars from Richard Mellon Scaife and other right-wing tycoons. Lies abounded, and the national press took it all seriously; the press "never caught up with the lies," writes Lewis. It never caught up, and thus played a vital role in giving credence to the fabrications.

Lewis is correct, I believe, in writing: "Most Americans understood that what was involved here was lying about sex—which goes with the territory and is not a reason for removing a president."

Lewis closes his essay by writing, "There is something profoundly disturbing about the way the haters were able to use the power of money and modern communications, in secret, to undo—nearly—our electoral process."

Especially disturbing, I think, is the willingness of most of the means of communication to play along with this effort at subverting the government and making it an open instrument of the most reactionary components of the possessors of wealth. The vileness

of this effort was thwarted, basically, by the common sense and decency of the majority of the people of the United States.

### **Washington and counterrevolution**

The defeats of fascism in Italy, Nazism in Germany, and monarchical imperialism in Japan made possible the Left-led revolutionary movements after the World War II throughout Europe, Latin America, Africa, and Asia. The governments in Washington and London devoted themselves to thwarting the success of these movements. In much of the world those repressive efforts succeeded, at the cost of hundreds of thousands of lives and the imposition of murderous regimes in Indonesia, the Middle East, Central America, South America, and Africa.

The latest in a series of exposures of such efforts has now surfaced from Iran. The 15 April 2000 *New York Times* refers in a front-page story to a CIA secret history revealing details of the conspiracy that brought Shah Pahlavi to the Iranian throne. He “ruled with an iron hand for another quarter of a century in close contact with the United States.” That “iron hand” meant the martyrdom of thousands. Much of the turmoil in Iran prior to the Shah’s return to power “had been stage-managed by CIA agents posing as communists.”

President Eisenhower, reports the *Times*, supported the counterrevolutionary conspiracy “because of fears about oil and communism.” The secret history of the coup “remains classified,” but some of those responsible have offered enough information to demonstrate that the British and U.S. governments “had almost complete contempt for the man they were empowering.”

This Iranian effort, the *Times* continues, “was a blueprint for a succession of CIA plots to foment coups and destabilize governments.” An instance was the overthrow of the democratic Arbenz government in Guatemala in 1954. But, the *Times* goes on, the Cuban intervention in 1961 was “disastrous,” and Castro’s leadership in thwarting this has earned him Washington’s continued hostility.

The basic documentation for all this “remains classified,” and “a number of the records of the operation” have been destroyed.

A very recent book further documents Washington's decisive role in bringing to power a murderous regime in Chile. This is Hugh O'Shaughnessy's *Pinochet: The Politics of Torture* (New York University Press, \$25.95). This demonstrates that Pinochet and his fellow thugs hunted down and murdered supporters of Allende's democratic government. The case of Orlando Letelier, killed in Washington, is well known; this book details the murders of additional Left opponents. Similar repressive regimes in South America slaughtered literally thousands of opponents of repression.

Efforts to open the files of agencies like the CIA have not been successful; official censorship has been openly defended. Indeed, George Tenet, the current director of the CIA, stated recently in a Senate hearing: "I would turn our gaze from the past. . . . It is dangerous, frankly, to keep looking over our shoulders" (quoted in the *Nation*, 22 May 2000, p. 28). Dangerous to whom, and for what reason? There is no good reason to believe that the policy of torture and murder has been repudiated, nor have the tormentors and murderers been punished.

### **Intensified social crisis**

The United States is experiencing a profound social crisis, as displayed in the marked and growing disparity of incomes within the population, and in the intensifying racist and elitist character of the so-called criminal justice system.

The facts, as jointly announced early in 2000 by the Center on Budget and Policy Priorities and Economic Policy Institute (both highly regarded research institutions in Washington, D.C.) are these:

The magnitude of the income disparities in the United States today and the degree to which disparity is increasing seems well beyond anything that is economically or socially desirable.

The Urban Institute, in a study released early this year, found that the income disparity induced "higher rates of child poverty." A table published in the 19 January 2000 *New York Times* (p. A21)

showed that from 1978 to 1998 average earnings in the nation (adjusted for inflation) for the poorest fifth of the population *fell* by six percent, and those for the second poorest fifth *fell* by one percent, while the income of the richest five percent *rose* by 55 percent.

As to prison population, the 15 February 2000 *Los Angeles Times*, citing the Justice Policy Institute of Washington, D.C., announced that there were currently two million men and women in prison (state, county, federal). This represented a doubling of the prison population in one decade, the equivalent of the growth made in the previous ninety years.

The study noted that “racial disparities are extreme—blacks are nearly *seven times* as likely to be incarcerated as whites,” and added that many African Americans “consider the prison system nothing short of a modern-day slave plantation.” It added that “although blacks compose about 13 percent of the population, they make up 50 percent of the state and federal prison population. . . . The odds that a black man will do time at some point in his life are 1-in-3; for whites, it is 1-in-25.”

The scandal of the execution of Black men as contrasted with white, and the certainty that many of those executed were innocent, has induced the Republican governor of Illinois (who favors the death penalty) to suspend all executions there pending further investigation.

Bob Herbert, writing in the *New York Times*, 4 February 2000, correctly observes that the “criminal justice system in this country is breaking down.” He documents awful injustices by police and courts not only in Chicago but also in Los Angeles and New York, and asks, “How many innocent people have been maimed or killed in the name of the law?”

The most notorious case of the class-based, racist nature of the prison system involves Mumia Abu-Jamal. The vindictiveness of the Pennsylvania judicial system in this case is extreme, no doubt motivated by the brilliance and courage of the particular victim. These characteristics are illustrated in the recently published collection of his writings, *All Things Censored* (New York: Seven Seas Press, \$25.95). This case reminds one of Angelo Herndon,

the Scottsboro Nine, and Angela Davis. In those cases, organized, dedicated, persistent struggle finally prevented judicial murder. Such efforts have so far halted the murder of Mumia Abu-Jamal. They must be continued and intensified. Mumia Abu-Jamal must be spared and vindicated in the name of humanity—not to mention decency.

### **Free Leonard Peltier!**

The rulers of the United States have compiled a record of atrocities unsurpassed by those of any other country. Rationalizing this has been racism—slavery; Jim Crow for generations of men, women, and children; and genocidal assaults upon the American Indian people.

Illustrating the latter is a contemporary crime: the case of the American Indian, Leonard Peltier, jailed for many years under barbarous conditions. This has been his punishment for daring to be a heroic and effective leader of his people seeking elementary justice.

The details are now available from his own pen in the splendid book edited by Harvey Arden, *Prison Writings*, with a fine preface by former Attorney General Ramsey Clark (Griffin, New York: St. Martin's Press, \$13.90).

I cannot believe that anyone reading this would not be moved to demand Peltier's freedom. Do immerse yourself in this cry from the heart—and then write to the president of the United States, urging him to free this man and undo this horrible stain upon the reputation of this nation.

[Editor's note: As we go to press, the list of outgoing President Clinton's first list of 62 pardons and commutations has just been released, and does not include the name of Leonard Peltier.]

## ABSTRACT

*József Szigeti*, “**The Principle of the Identity of Identity and Not-Identity**”—The identity of identity and not-identity is one of the basic laws of Hegelian dialectics. The author analyzes some main aspects of this basic law in the mirror of the history of ideas and recent research.

## ABREGE

*József Szigeti*, « **Le Principe de l’identité de l’identité et non-identité** » – L’identité de l’identité et non-identité est une des lois fondamentales de la dialectique hégélienne. L’auteur analyse quelques aspects principaux de cette loi fondamentale comme réfléchis dans l’histoire d’idées et dans les recherches récentes.